Quality Control and Reliability
HANDBOOK (Interim) **H 108**

SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY TESTING
(Based on Exponential Distribution)

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OFFICE OF THE ASSISTANT SECRETARY OF DEFENSE
(Supply and Logistics)
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SUPPLY AND LOGISTICS

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This handbook (interim) was developed by the Bureau of Naval Weapons, Department of the Navy on behalf of the Office of the Assistant Secretary of Defense (Supply and Logistics). It was prepared to meet a growing need for the use of standard sampling plans for life and reliability testing. Such plans may be used to demonstrate the conformance of equipments, subassemblies and component parts to established reliability requirements.

Suggestions for improvements of this handbook should be addressed to the Office of the Assistant Secretary of Defense (Supply and Logistics), Washington 25, D.C.

FOREWORD

This handbook has been prepared to meet a growing need for the use of standard sampling procedures and tables for life and reliability testing in Government procurement, supply, and maintenance quality control operations as well as in research and development activities where applicable.

A characteristic feature of most life tests is that the observations are ordered in time to failure. If, for example, 20 radio tubes are placed on life test, and t_i denotes the time when the i'th tube fails, the data occur in such a way that $t_1 \leqq t_2 \leqq \ldots \leqq t_{20}$. The same kind of ordered observations will occur whether the problem under consideration deals with the life of electric bulbs, the life of electronic components, the life of ball bearings, or the length of life of human beings after they are treated for a disease. The examples just given all involved ordering in time.

In destructive testing involving such situations as the current needed to blow a fuse, the voltage needed to break down a condenser, the force needed to rupture a physical material, the test can often be arranged in such a way that every item in the sample is subjected to precisely the same stimulus (current, voltage, stress). If this is done, then clearly the weakest item will be observed to fail first, the second weakest next, etc. While the random variable considered mostly in this handbook is time to failure, it should be emphasized, however, that the methodology provided herein can be adopted to the testing situations mentioned above where the random variable is current, voltage, stress, etc.

Chapter 1 of the handbook describes general procedures and definitions of terms used in life test sampling. Chapter 2 describes specific procedures and applications of the life test sampling plans for determining conformance to established reliability requirements.

An explanation of the system used in numbering the pages and paragraphs in this handbook is appropriate. Each chapter carries its own independent sequence of page, paragraph, and table numbers. Pages are numbered as follows: the figure preceding the decimal point represents the number of the chapter and the numbers following the decimal point denote the page within the chapter. Thus, page 2.16 means chapter 2, page 16. Paragraphs are numbered so that the first figure represents the number of the chapter, the uppercase letter represents the section within the chapter, and finally the paragraph number itself For example, 2A1.5 means chapter 2, section A, and paragraph 1.5. Tables and graphs are numbered with the first figure representing the number of the chapter, the uppercase letter representing the section, and finally the table or figure number itself. Thus, table 2C–5 means chapter 2, section C, table 5, and fig. 2D–3 means chapter 2, section D, figure 3.

Whenever the methodology or choice of procedures in the handbook require clarification, the user is advised to consult a qualified mathematical statistician and reference should be made to appropriate technical reports and other publications in the field.

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INTRODUCTION

The theory underlying the development of the life test sampling plans and procedures of chapter 2 of this handbook assume the measurements of the length of life are drawn from an exponential distribution. Statistical test procedures for determining the validity of the exponential distribution assumption have appeared in the technical statistical journals. Professor Benjamin Epstein has published a comprehensive paper (in two parts) on this subject in the February and May 1960 issues of "Technometrics."* Part I of the paper contains descriptions of the mathematical and graphical procedures as well as an extensive bibliography for reference purposes. Numerical examples illustrating the statistical procedures are included in Part II of the paper.

*B. Epstein, "Tests for the Validity of the Assumption that the Underlying Distribution of Life is Exponential," *Technometrics*, Feb. 1960 and May 1960.

GENERAL DEFINITIONS OF LIFE AND RELIABILITY TEST TERMS

1A1 SCOPE

1A1.1 Purpose. Chapter 1 of this handbook provides definitions of terms required for the use of the life test sampling plans and procedures of chapter 2.

1A1.2 Life Test. Life test is the process of placing the "unit of product" under a specified set of test conditions and measuring the time it takes till failure.

1A1.3 Unit of Product. The unit of product is the entity of product which may be placed on life test.

1A1.4 Specifying Failure. The state that constitutes a failure must be specified in advance of the life test.

1A1.5 Life Test Sampling Plan. A life test sampling plan is a procedure which specifies the number of units of product from a lot which are to be tested, and the criterion for determining acceptability of the lot.

1A1.6 Life Test Terminated Upon Occurrence of Preassigned Number of Failures. Life test sampling plans whereby testing is terminated when a preassigned termination number of failures, r, occur are given in section 2B of this handbook.

1A1.7 Life Test Terminated at Preassigned Time. Life test sampling plans whereby testing is terminated when a preassigned termination time, T, is reached are given in section 2C of this handbook.

1A1.8 Sequential Life Test. Sequential life test is a life test sampling plan whereby neither the number of failures nor the time required to reach a decision are fixed in advance but instead decisions depend on the accumulated results of the life test. Information on the observed time to failure are accumulated over time and the results at any time determine the choice of one among three possible decisions: (1) the lot meets the acceptability criterion, (2) the lot does not meet the acceptability criterion, or (3) the evidence is insufficient for either decision (1) or (2) and the test must continue. Sequential life test sampling plans are given in section 2D of this handbook and have the advantage over the life test sampling plans mentioned in paragraphs 1A1.6 and 1A1.7 in that, for the same operating characteristic curve, the expected waiting time and the expected number of failures required to reach a decision as to lot acceptability are less for the sequential life tests.

1A1.9 Expected Number of Failures. The number of failures required for decision is the number of failures that have occurred at the time the decision as to lot acceptability is reached. For the life test sampling plans mentioned in paragraph 1A1.6, this number of failures is known in advance of the life test; but, for the sampling plans mentioned in paragraphs 1A1.7 and 1A1.8, this number cannot be predetermined. The expected number of failures required for decision is the average of the number of failures required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected number of failures can be predetermined for the sampling plans mentioned in paragraphs 1A1.6, 1A1.7, and 1A1.8.

1A1.10 Expected Waiting Time. The waiting time required for decision is the time elapsed from the start of the life test to the time decision is reached as to lot acceptability. The waiting time required for decision cannot be predetermined for any of the sampling plans mentioned in paragraph 1A1.6, 1A1.7, or 1A1.8. The expected waiting time required for decision is the average of the waiting times required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected waiting time can be predetermined for the sampling plans mentioned in paragraphs 1A1.6, 1A1.7, and 1A1.8.

1A2 LENGTH OF LIFE

1A2.1 Length of Life. The terms "length of life" and "time to failure"may be used interchangeably and shall denote the length of time it takes for a unit of product to fail after being placed on life test. The length of time may be expressed in any convenient time scale such as seconds, hours, days, etc.

1A2.2 Mean Time to Failure. The terms "mean time to failure" and "mean life" may be used interchangeably and shall denote the mean (or equivalently, the average) length of life of items in the lot. Mean life is denoted by θ.

1A2.3 Acceptable Mean Life. The acceptable mean life, θ_0, is the minimum mean time to failure which is considered satisfactory.

1A2.4 Unacceptable Mean Life. The unacceptable mean life, θ_1, $(\theta_1 < \theta_0)$, is the mean time to failure such that lots having mean life less than or equal to θ_1 are considered unsatisfactory. The interval between θ_0 and θ_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the mean life decreases from θ_0 to θ_1.

1A3 FAILURE RATE

1A3.1 Proportion of Lot Failing Before Specified Time. The term "proportion of lot failing before specified time," p, denotes the fraction of the lot that fails before some specified time T, i.e.,

$$p = 1 - exp(-T/\theta).$$

1A3.2 Failure Rate During Period of Time. The "failure rate during period of time T," C, is given by—

$$G = \frac{1}{T}\left\{ 1 - exp\,(T/\theta) \right\} = p/T.$$

1A3.3 Instantaneous Failure Rate. The "instantaneous failure rate" or "hazard rate" is given by—

$$Z = 1/\theta.$$

1A3.4 Acceptable Proportion of Lot Failing Before Specified Time. The "acceptable proportion of lot failing before specified time," p_0, is the maximum fraction of the lot that may fail before the specified time T and still result in the lot being considered satisfactory.

1A3.5 Unacceptable Proportion of Lot Failing Before Specified Time. The "unacceptable proportion of lot failing before specified time," p_1, $(p_1 > p_0)$, is the minimum fraction of the lot that may fail before time T and results in the lot being considered unsatisfactory. The interval between p_0 and p_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the fraction of the lot failing before time T increases from p_0 to p_1.

1A3.6 Acceptable Failure Rate During Period of Time. The "acceptable failure rate during period of time," G_0, is the maximum failure rate during period of time that can be considered satisfactory.

1A3.7 Unacceptable Failure Rate During Period of Time. The "unacceptable failure rate during period of time," G_1, $(G_1 > G_0)$, is the minimum failure rate during period of time that results in the lot being considered unsatisfactory. The interval between G_0 and G_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the failure rate increases from G_0 to G_1.

1A3.8 Life Test Sampling Plans Based on Failure Rates. Life test sampling plans which are based on failure rates are given in section 2C, part III.

1A4 OPERATING CHARACTERISTIC CURVES AND SAMPLING RISKS

1A4.1 Operating Characteristic Curves. The operating characteristic (OC) curve of a life test sampling plan is the curve which shows the probability that a submitted lot with given mean life would meet the acceptability criterion on the basis of that sampling plan.

1A4.2 Producer's Risk. The producer's risk, α, is the probability of rejecting lots with mean life θ_0. For the procedures of section 2C, part III, the producer's risk may also be defined as the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.

1A4.3 Consumer's Risk. The consumer's risk, β, is the probability of accepting lots with mean life θ_1. For the procedures of section 2C, part III, the consumer's risk may also be defined as the probability of accepting lots with unacceptable proportion of lot failing before specified time, p_1.

1A5 SUBMITTAL OF PRODUCT

1A5.1 Lot. The term "lot" shall mean either an "inspection lot", i.e., a collection of units of product, manufactured under essentially the same conditions, from which a sample is drawn and tested to determine compliance with the acceptability criterion; or, a "preproduction lot", i.e., one or more units of product submitted prior to initiation of production for test to determine compliance with the acceptability criterion.

1A6 SAMPLE SELECTION

1A6.1 Drawing of Samples. A sample is one or more units of product drawn at random from a lot.

1A6.2 Testing Without Replacement. Life test sampling without replacement is a life test procedure whereby failed units are not replaced.

1A6.3 Testing With Replacement. Life test sampling with replacement is a life test procedure whereby the life test is continued with each failed unit of product replaced by a new one, drawn at random from the same lot, as soon as the failure occurred. In the case of complex unit of product, this may be interpreted to mean replacement of the component which caused the failure by a new component drawn at random from the same lot of components. When the "sample sizes" are the same in both instances, the expected waiting time required for decision when testing with replacement is less than when testing without replacement.

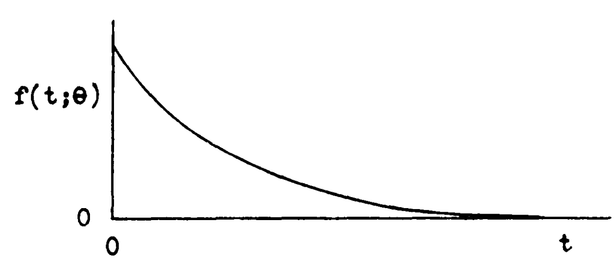
1A6.4 Sample Size. The sample size, n, for a life test is the number of units of product placed on test at the start of a life test. When testing with replacement, the total number of units of product placed on test will, in general, be greater than the original sample size. The sample sizes, for the life test plans of chapter 2, depend on the relative cost of placing large numbers of units of product on test and on the expected length of time the life tests must continue in order to determine acceptability of the lots. Increasing the sample size will, on one hand, cut the average time required to determine acceptability but on the other hand will increase the cost due to placing more units of product on test.

1A7 EXPONENTIAL DISTRIBUTION

1A7.1 Exponential Distribution With One Parameter. The density function for the exponential distribution with one parameter is given by—

$$f(t;\theta)=1/\theta exp(-t/\theta) \qquad t\geq 0,\ \theta>0$$
$$=0 \qquad t<0$$

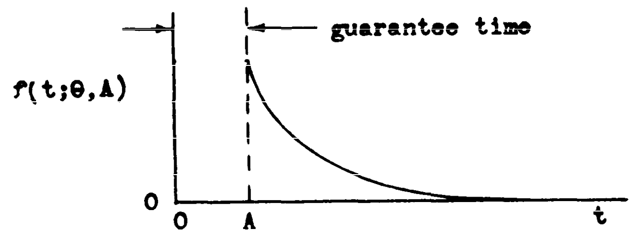
and the function has the following general graphical form:



1A7.2 Exponential Distribution With Two Parameters. The density function for the exponential distribution with two parameters is given by—

$$f(t;\theta,A) = 1/\theta \; exp[-(t-A)/\theta] \qquad t \geq A \geq 0$$
$$= 0 \qquad \text{elsewhere}$$

and the function has the following general graphical form:



The quantity A is called "guarantee time" and the one parameter case is a special case of the two parameter distribution with a guarantee time of zero.

1A7.3 Exponential Distribution When Number of Parameters Is Unspecified. In this handbook, whenever the term "exponential distribution" is mentioned without specific mention of the number of parameters, it shall be assumed to mean the exponential distribution with one parameter.

CHAPTER 2

SAMPLING PROCEDURES AND TABLES

(Exponential Distribution)

INTRODUCTION

The theory underlying the development of the life test sampling plans of this chapter, including the operating characteristic curves, assume that the measurements of the length of life are drawn from an exponential distribution.

It is important to note that the life test sampling plans of this chapter are not to be used indiscriminately, simply because it is possible to obtain life test data. Only after the exponential assumption is deemed reasonable should the sampling plans of this chapter be used.

This chapter is divided into four sections. Section A describes general procedures and description of life test sampling plans. Section B describes specific procedures and applications of sampling plans when life tests are terminated upon the occurrence of a preassigned number of failures and section C provides sampling plans when life tests are terminated at a preassigned time. Section D describes sequential life test sampling plans.

Section B is divided into three parts: (1) Acceptance Procedures; (2) Expected Duration of Life Tests and Cost Considerations in Selection of Sample Sizes; and (3) Life Test Plans for Certain Specified Values of α, β, and θ_1/θ_0. Section C is also divided into three parts: (1) Acceptance Procedures; (2) Life Test Plans for Certain Specified Values of α, β, θ_1/θ_0, and T/θ_0; and (3) Life Test Plans Based on Proportion of Lot Failing Before Specified Time. Section D consists of only one part.

Operating characteristic (OC) curves for the life test sampling plans of section 2B, part I; section 2C, part I; and section 2D are shown in table 2A-2 for the corresponding sampling plans in these sections were matched with respect to their OC curves. The operating characteristic curves in table 2A-2 have been computed for the life test sampling plans of section 2B, part I, but are equally applicable for the sampling plans of section 2C, part I, and section 2D.

The procedures of this chapter are based on the premise that the life tests are monitored continuously. If the tests are monitored only periodically, the values obtained from the tables and curves are only approximations.

SECTION 2A

GENERAL DESCRIPTION OF LIFE TEST SAMPLING PLANS

2A1 SCOPE

2A1.1 Purpose. Chapter 2 of this handbook establishes life test sampling plans for determining acceptability of a product when samples are drawn at random from an exponential distribution.

2A1.2 Specifying Acceptable Mean Life. Before the start of the life test, the particular value of the acceptable mean life, θ_0, shall be specified except when using the procedures of section 2C, part III.

2A1.3 Specifying Unacceptable Mean Life. The particular value of the unacceptable mean life, θ_1, shall be specified in advance of the life test when using the life test procedures of section 2B, part III, and section 2C, part II.

2A1.4 Specifying Acceptable Proportion of Lot Failing Before Specified Time. The particular value, p_0, of the acceptable proportion of lot failing before specified time to be used in the life test shall be specified in advance for the procedures of section 2C, part III.

2A1.5 Specifying Unacceptable Proportion of Lot Failing Before Specified Time. The particular value, p_1, of the unacceptable proportion of lot failing before specified time shall be specified in advance of the life test when using the procedures of section 2C, part III.

2A2 SAMPLING RISKS

2A2.1 Producer's Risk. The producer's risk, α, is the probability of rejecting lots with mean life θ_0. For the procedures of section 2C, part III, the producer's risk may also be defined as the probability of rejecting lots with p_0 as the acceptable proportion of lot failing before specified time. Summarized below are the various numerical values of α and the master sampling tables in which they are given.

Procedures for—	*Producer's risk*	*Table*
Section 2B, part I	0.01, 0.05, 0.10, 0.25, 0.50	2B-1
Section 2B, part III	.01, .05, .10, .25	2B-5
Section 2C, part I	.01, .05, .10, .25, .50	2C-1, 2C-2
Section 2C, part II	.01, .05, .10, .25	2C-3, 2C-4
Section 2C, part III	.01, .05, .10	2C-5
Section 2D	.01, .05, .10, .25, .50	2D-1

2A2.2 Specifying Producer's Risk. The particular value of α to be used in the life test shall be selected from among those given in paragraph 2A2.1 and specified in advance of the life test.

2A2.3 Consumer's Risk. The consumer's risk, β, is the probability of accepting lots with mean life θ_1. For the procedures of section 2C, part III, the consumer's risk may also be defined as the probability of accepting lots with p_1 as the unacceptable proportion of lot failing before specified time. Summarized below are the various numerical values of β and the master sampling tables in which they are given.

Procedures for—	*Consumer's risk*	*Table*
Section 2B, part I	.10	2B-1
Section 2B, part III	.01, .05, .10, .25	2B-5
Section 2C, part I	.10	2C-1, 2C-2
Section 2C, part II	.01, .05, .10, .25	2C-3, 2C-4
Section 2C, part III	.01, .05, .10	2C-5
Section 2D	.10	2D-1

The smaller the value of β, the greater is the protection against acceptance of lots with low mean life or high failure rate.

2A2.4 Specifying Consumer's Risk. The particular value of β to be used in the life test shall be selected from among those given in paragraph 2A2.3 and specified in advance of the life test.

2A3 OPERATING CHARACTERISTIC CURVES

2A3.1 Operating Characteristic Curve. The operating characteristic (OC) curve of a life test sampling plan is the curve which shows the probability that a submitted lot with given mean life would meet the acceptability criterion on the basis of that sampling plan. The OC curves given in table 2A-2 are equally applicable for the sampling plans of section 2B, part I; section 2C, part I; and section 2D of this chapter. Moreover, the OC curves are also equally applicable for both the sampling with and without replacement procedures. The abscissas of the OC curves are expressed as the ratio θ/θ_0 in table 2A-2 so that the same set of OC curves is applicable regardless of the value of the specified acceptable mean life θ_0.

2A3.2 Sampling Plan Code Designation. The life test sampling plans of section 2B, part I; section 2C, part I; and section 2D, along with their associated OC curves, are designated by code letters and numbers. The sample code is given in table 2A-1 and is determined by the values of α, β, and θ_1/θ_0. The OC curves of all sampling plans designated by the same code pass through the two points $(1, 1-\alpha)$ and $(\theta_1/\theta_0, \beta=0.10)$. Thus, all sampling plans which are designated by the same code offer essentially the same protection.

2A3.3 The Ratio θ_1/θ_0 as Measure of Protection Offered by Sampling Plan. The consumer's risk β has been defined in paragraph 2A2.3 as the risk of accepting lots with mean life θ_1. Because the OC curves are drawn with abscissa θ/θ_0, the ratio θ_1/θ_0 is also a measure of mean life which is accepted with probability β. The ratio θ_1/θ_0 must be greater than zero but less than unity. If α, β, θ_0 are kept constant, as θ_1/θ_0 increases, the protection offered by the sampling plan against accepting lots with low mean life also increases. Thus table 2A-1 allows comparisons in the amount of protection offered by the various sampling plans, for in any column, the protection increases as θ_1/θ_0 increases.

Table 2A-1

Life Test Sampling Plan Code Designation

Index of Life Test Sampling Plans of section 2B, part I; section 2C, part I; and section 2D. OC curves are given for these sampling plans in table 2A-2

$\alpha=0.01$ $\beta=0.10$		$\alpha=0.05$ $\beta=0.10$		$\alpha=0.10$ $\beta=0.10$		$\alpha=0.25$ $\beta=0.10$		$\alpha=0.50$ $\beta=0.10$	
Code	θ_1/θ_0	Code	θ_1/θ_0	Code	θ_1/θ_0	Code	θ_1/θ_0	Code	θ_1/θ_0
A-1	0.004	B-1	0.022	C-1	0.046	D-1	0.125	E-1	0.301
A-2	.038	B-2	.091	C-2	.137	D-2	.247	E-2	.432
A-3	.082	B-3	.154	C-3	.207	D-3	.325	E-3	.502
A-4	.123	B-4	.205	C-4	.261	D-4	.379	E-4	.550
A-5	.160	B-5	.246	C-5	.304	D-5	.421	E-5	.584
A-6	.193	B-6	.282	C-6	.340	D-6	.455	E-6	.611
A-7	.221	B-7	.312	C-7	.370	D-7	.483	E-7	.633
A-8	.247	B-8	.338	C-8	.396	D-8	.506	E-8	.652
A-9	.270	B-9	.361	C-9	.418	D-9	.526	E-9	.667
A-10	.291	B-10	.382	C-10	.438	D-10	.544	E-10	.681
A-11	.371	B-11	.459	C-11	.512	D-11	.608	E-11	.729
A-12	.428	B-12	.512	C-12	.561	D-12	.650	E-12	.759
A-13	.470	B-13	.550	C-13	.597	D-13	.680	E-13	.781
A-14	.504	B-14	.581	C-14	.624	D-14	.703	E-14	.798
A-15	.554	B-15	.625	C-15	.666	D-15	.737	E-15	.821
A-16	.591	B-16	.658	C-16	.695	D-16	.761	E-16	.838
A-17	.653	B-17	.711	C-17	.743	D-17	.800	E-17	.865
A-18	.692	B-18	.745	C-18	.774	D-18	.824	E-18	.882

Producer's risk α is the probability of rejecting lots with mean life θ_0.
Consumer's risk β is the probability of accepting lots with mean life θ_1.

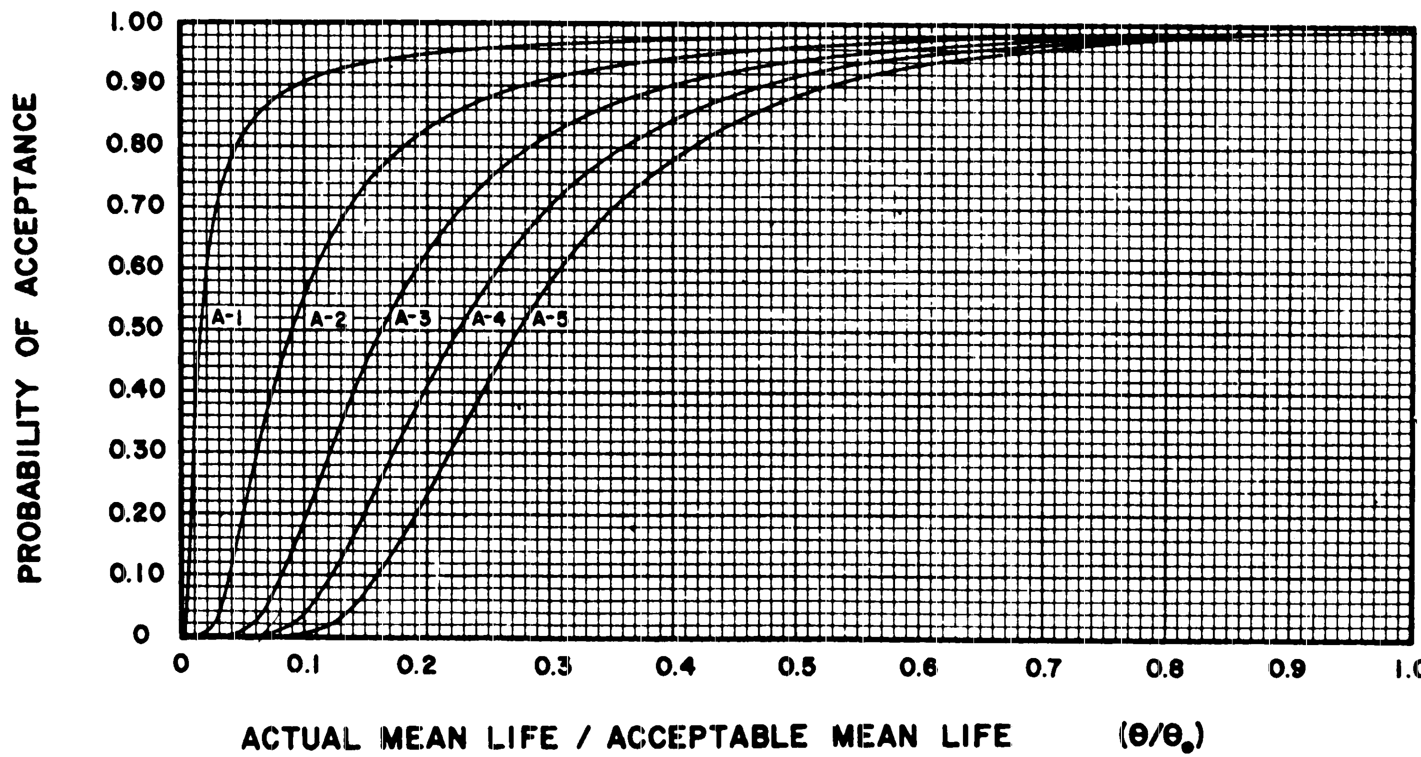
Table 2A-2

Operating Characteristic Curves for Life Test Sampling Plans of section 2B, part I; section 2C, part I; and section 2D

TABLE 2A-2

OPERATING CHARACTERISTIC CURVES FOR LIFE TESTS TERMINATED UPON OCCURRENCE OF PRE-ASSIGNED NUMBER OF FAILURES

(Curves for sequential plans and tests terminated at pre-assigned time are essentially equivalent)



These curves are valid only when sampling from an exponential distribution.

Note: Notations on curves are sample size.

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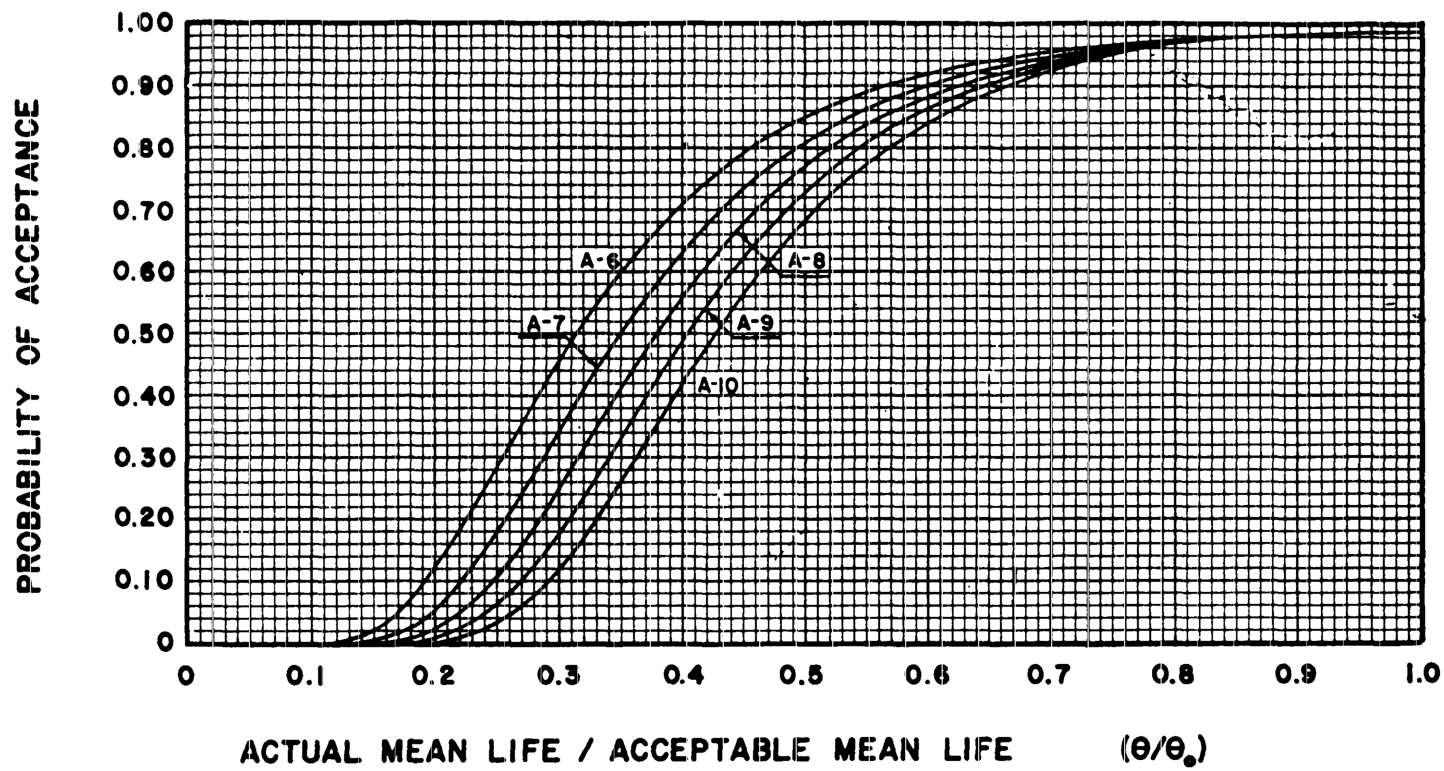

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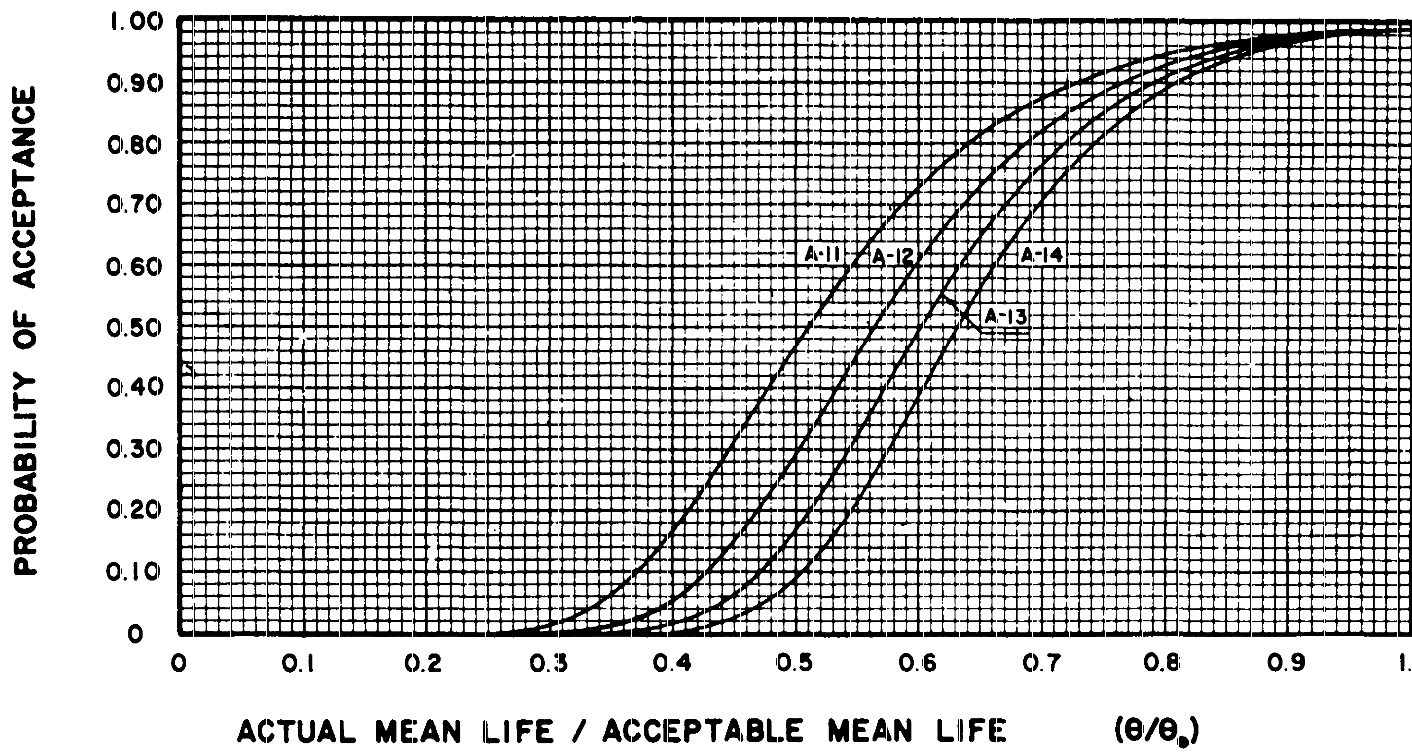

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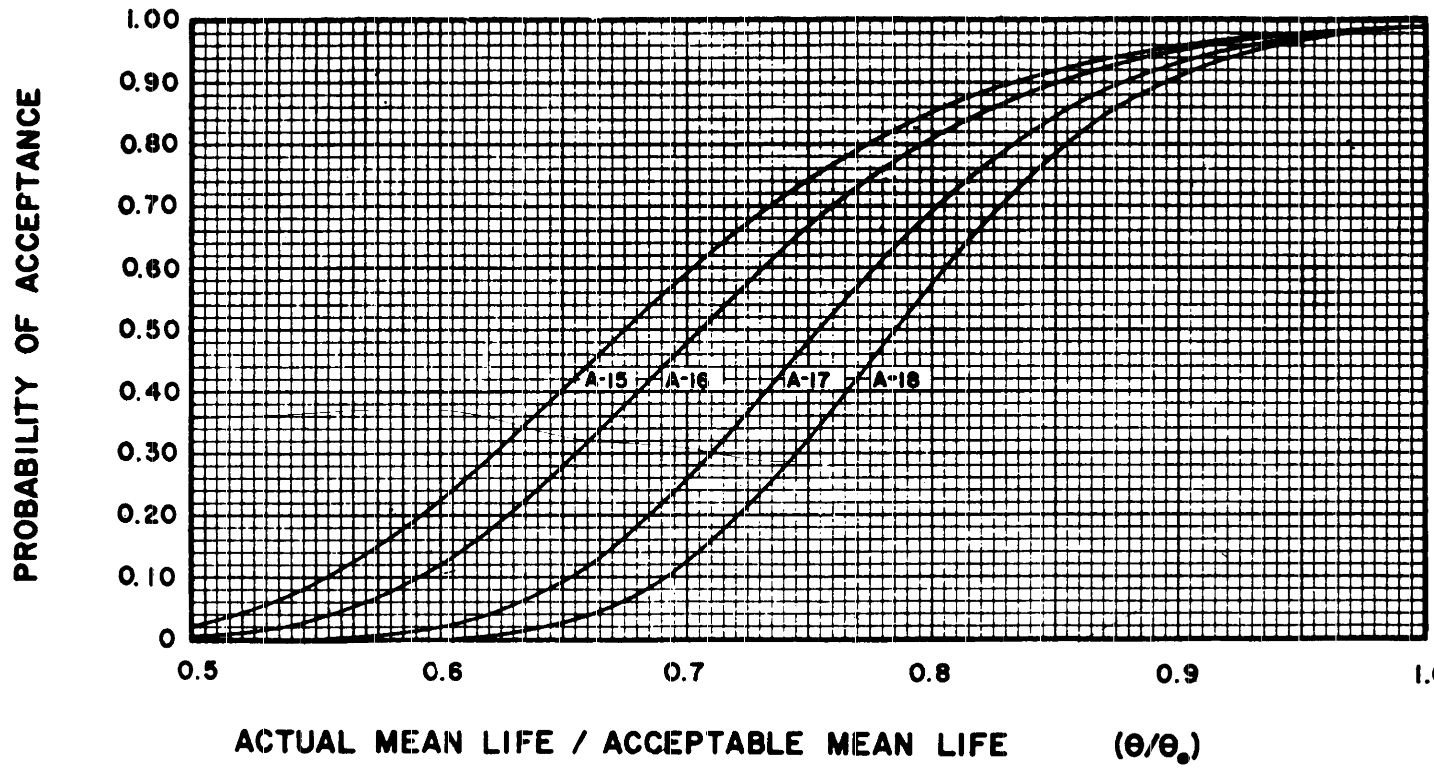

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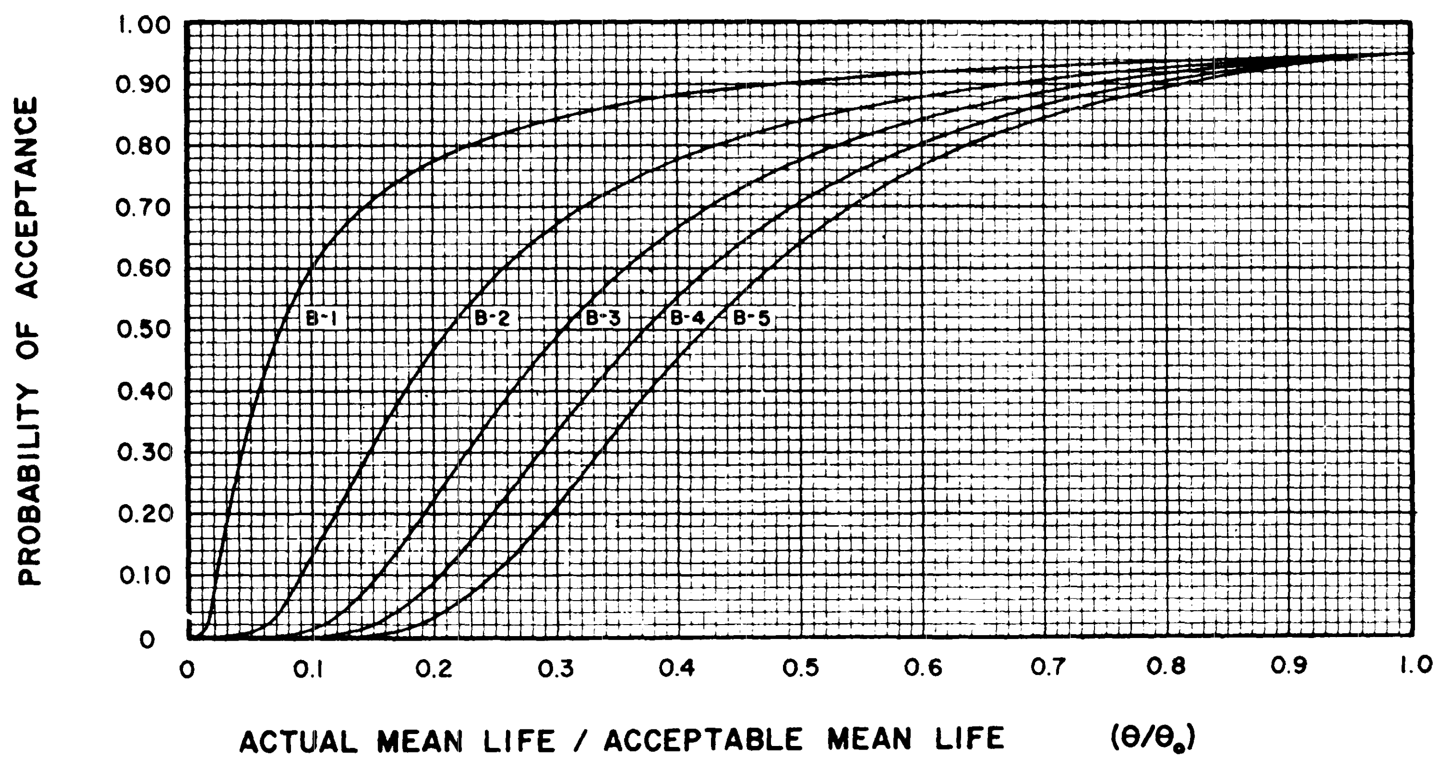

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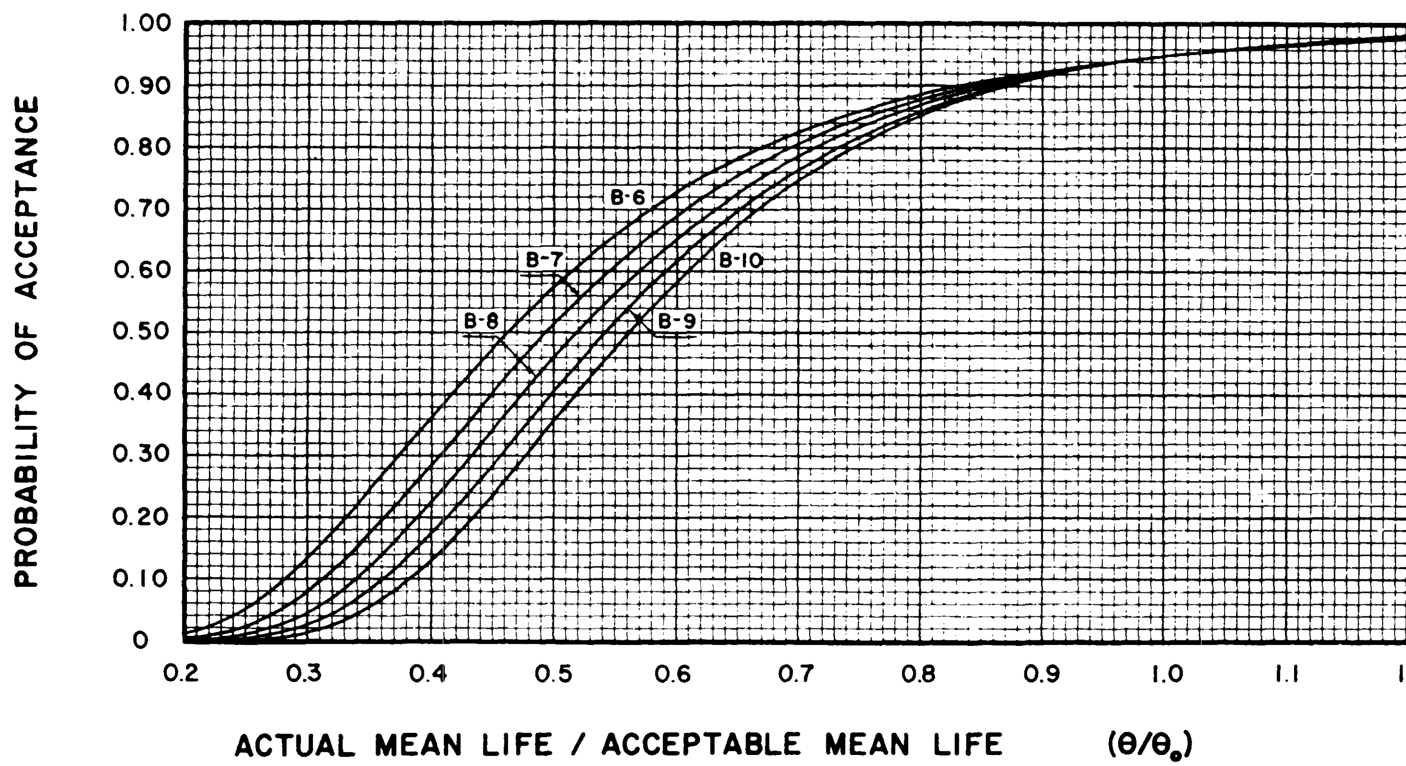

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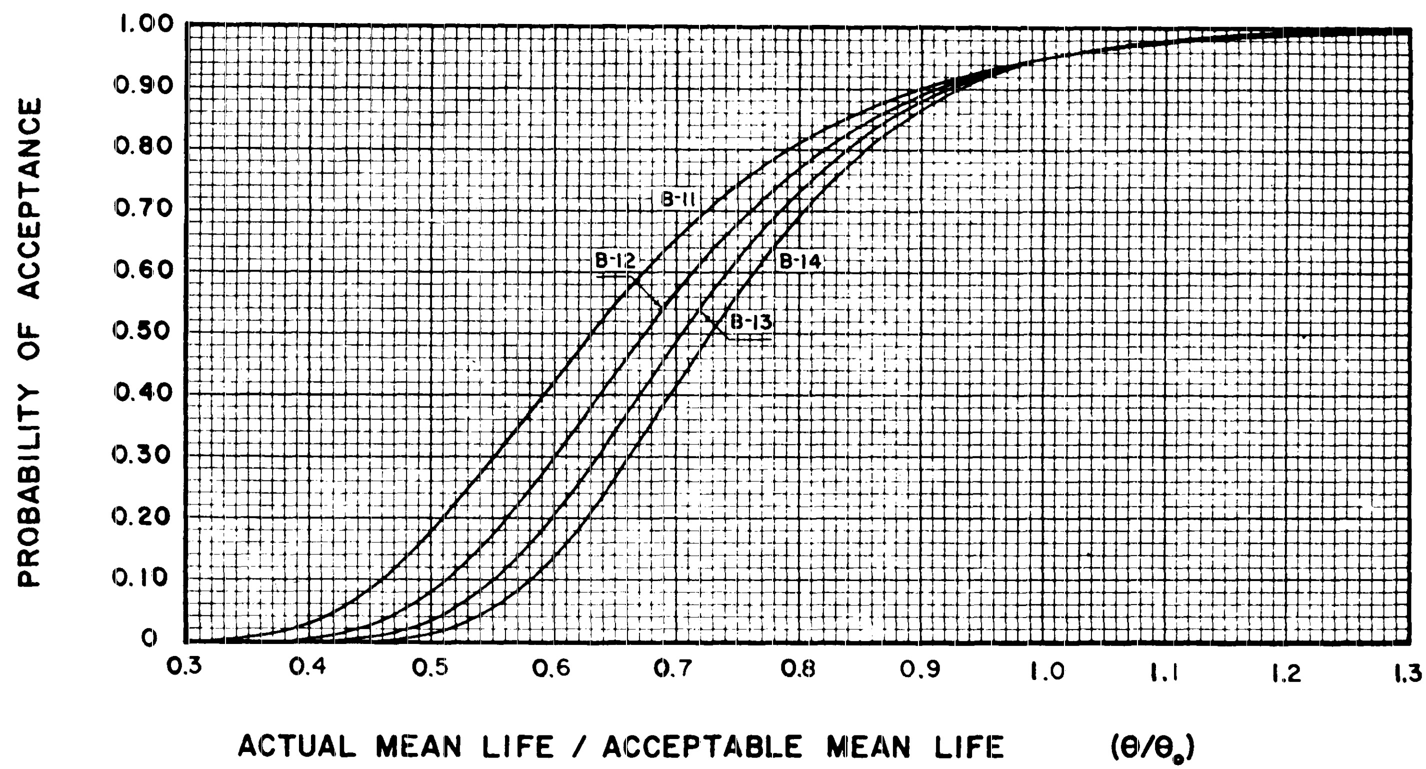

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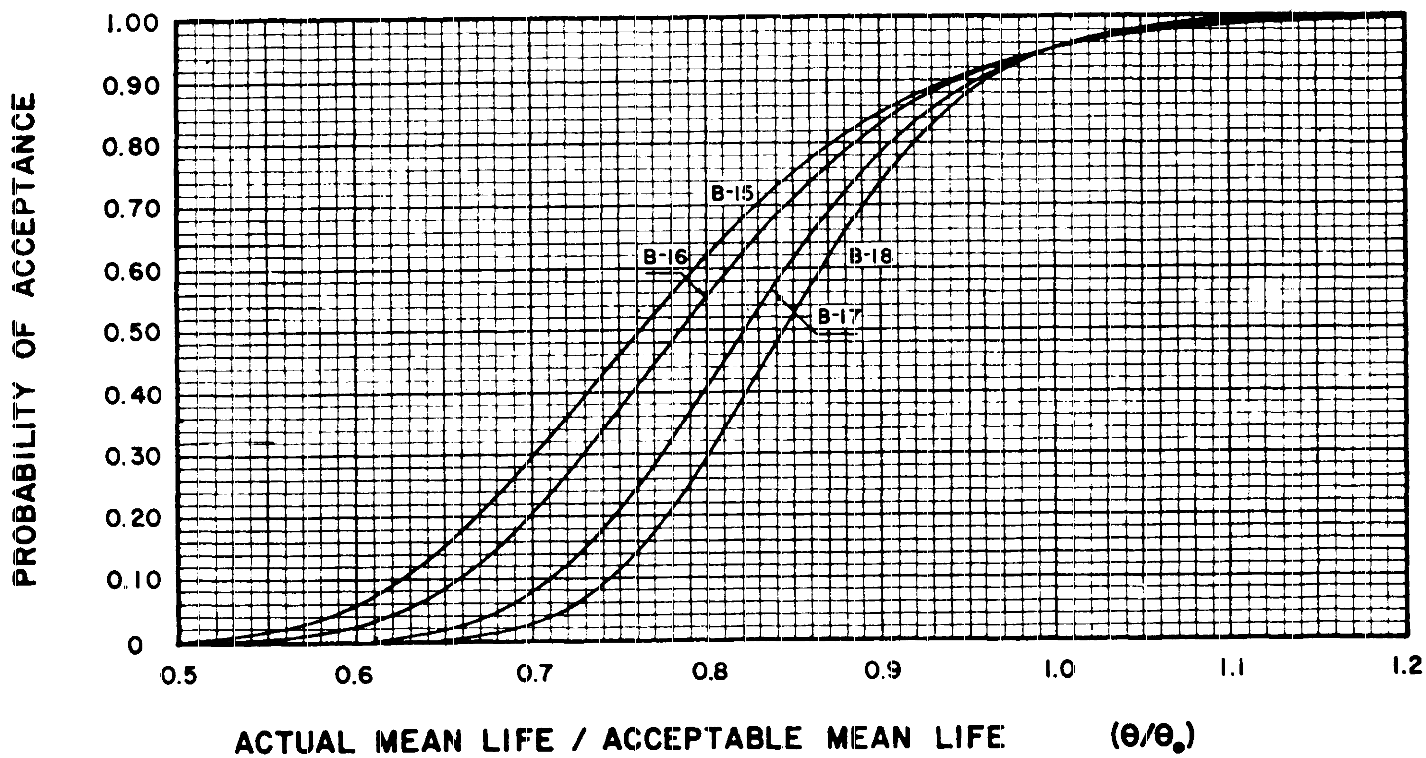


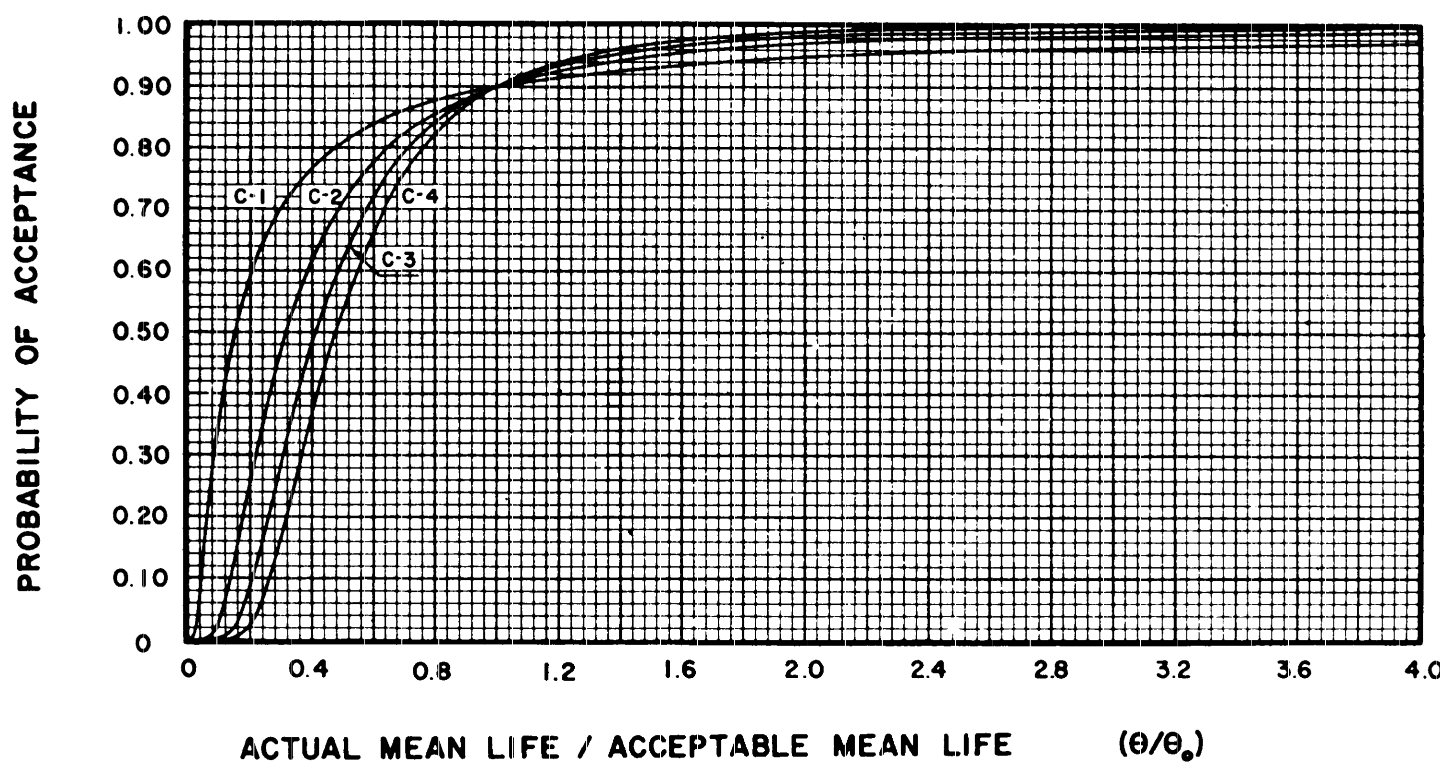
These curves are valid only when sampling from an exponential distribution.

Note: Notations on curves are sample plan code designation

TABLE 2A-2

OPERATING CHARACTERISTIC CURVES FOR LIFE TESTS TERMINATED UPON OCCURRENCE OF PRE-ASSIGNED NUMBER OF FAILURES

(Curves for sequential plans and tests terminated at pre-assigned time are essentially equivalent)



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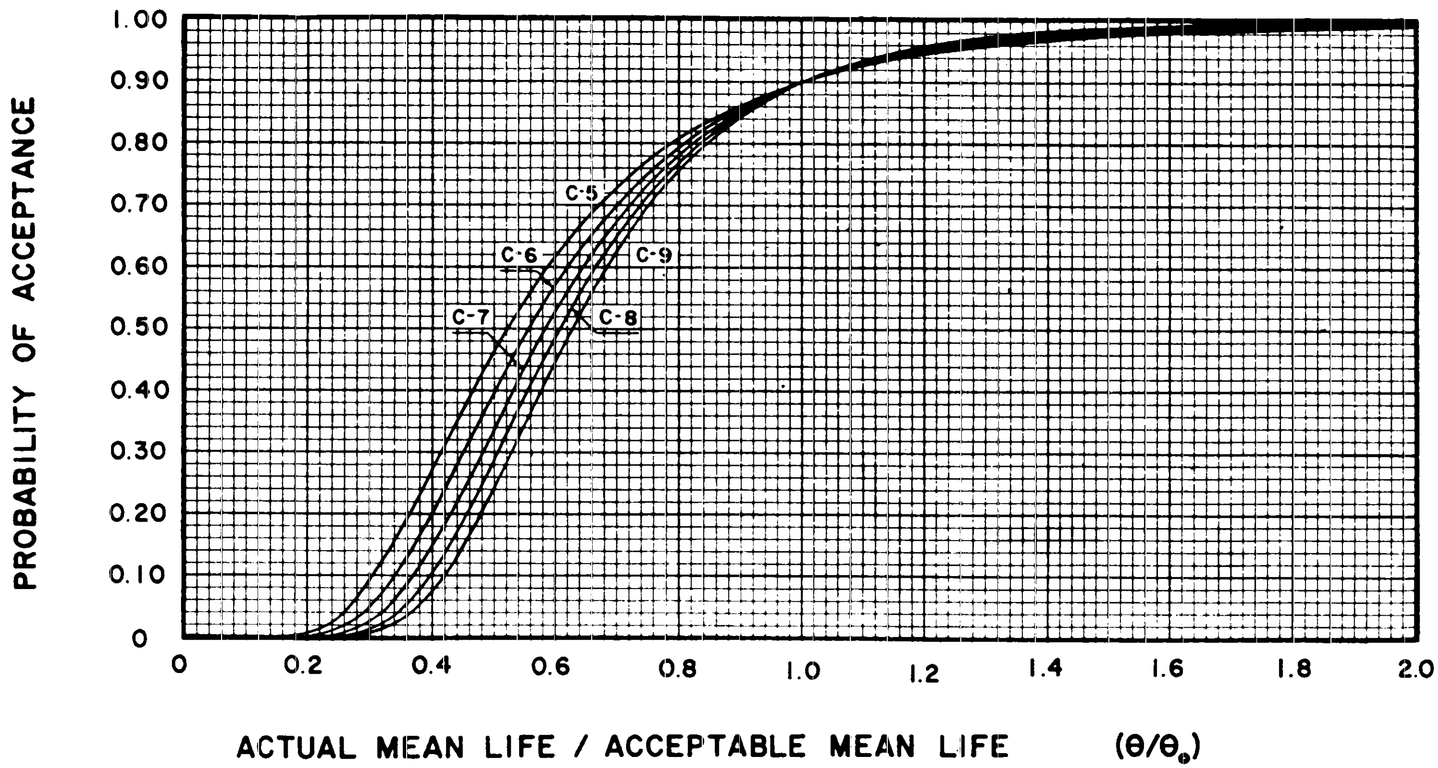

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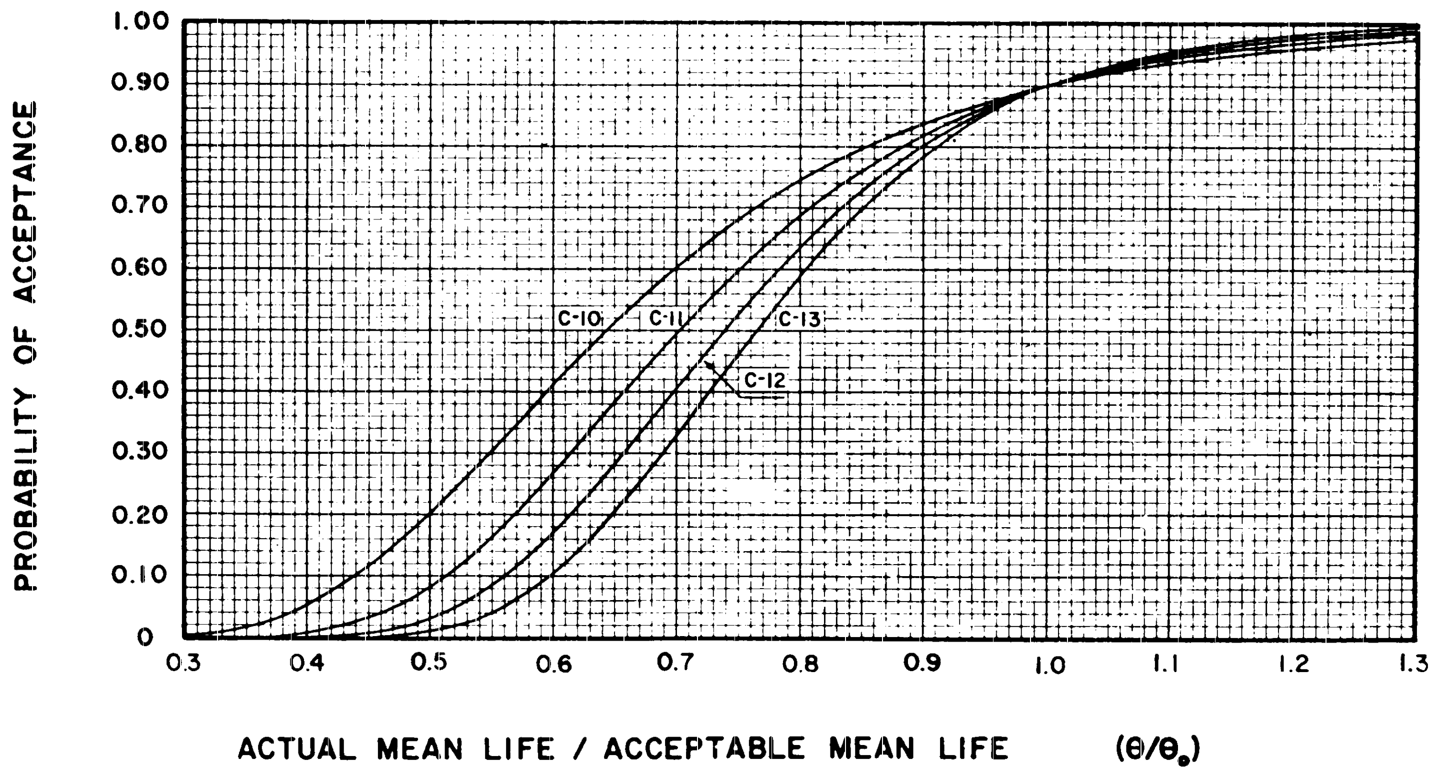

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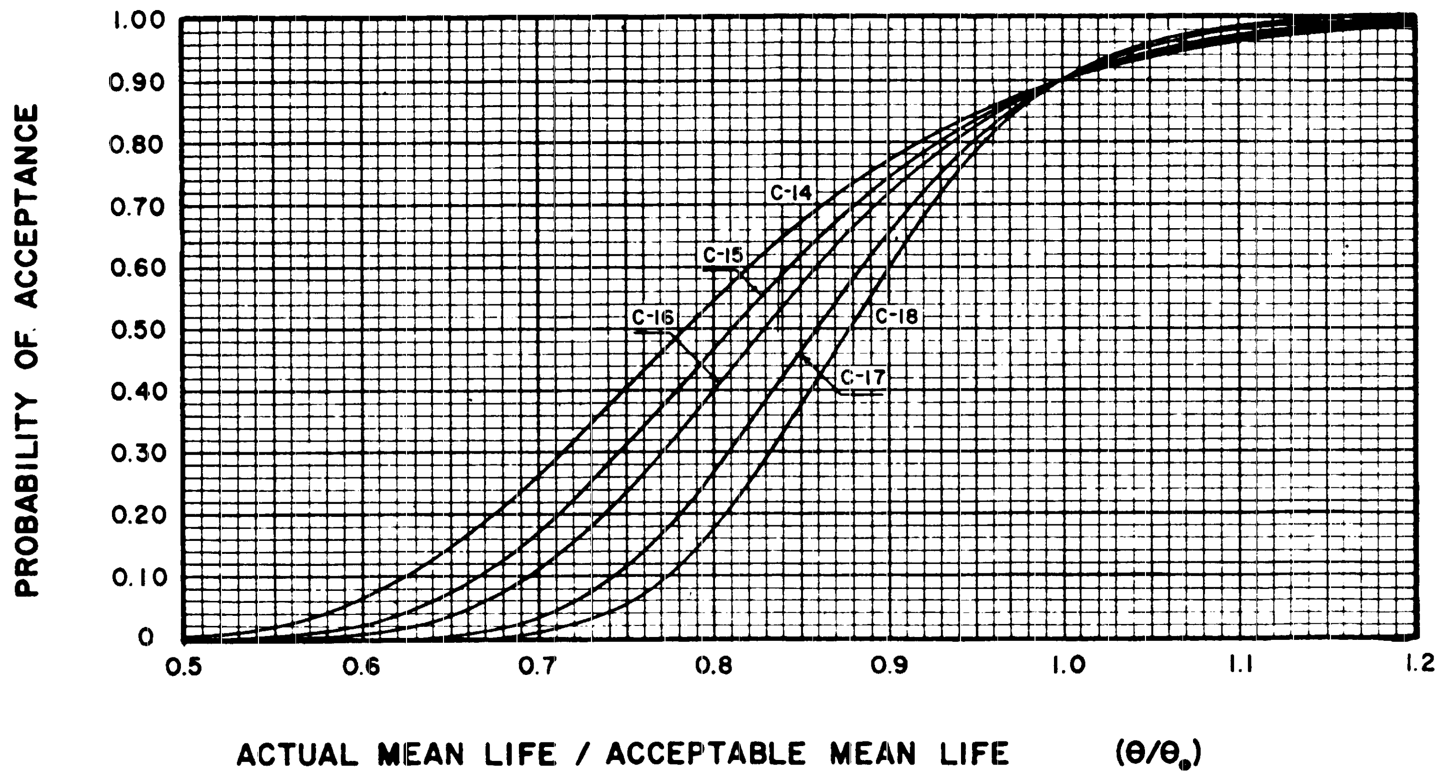

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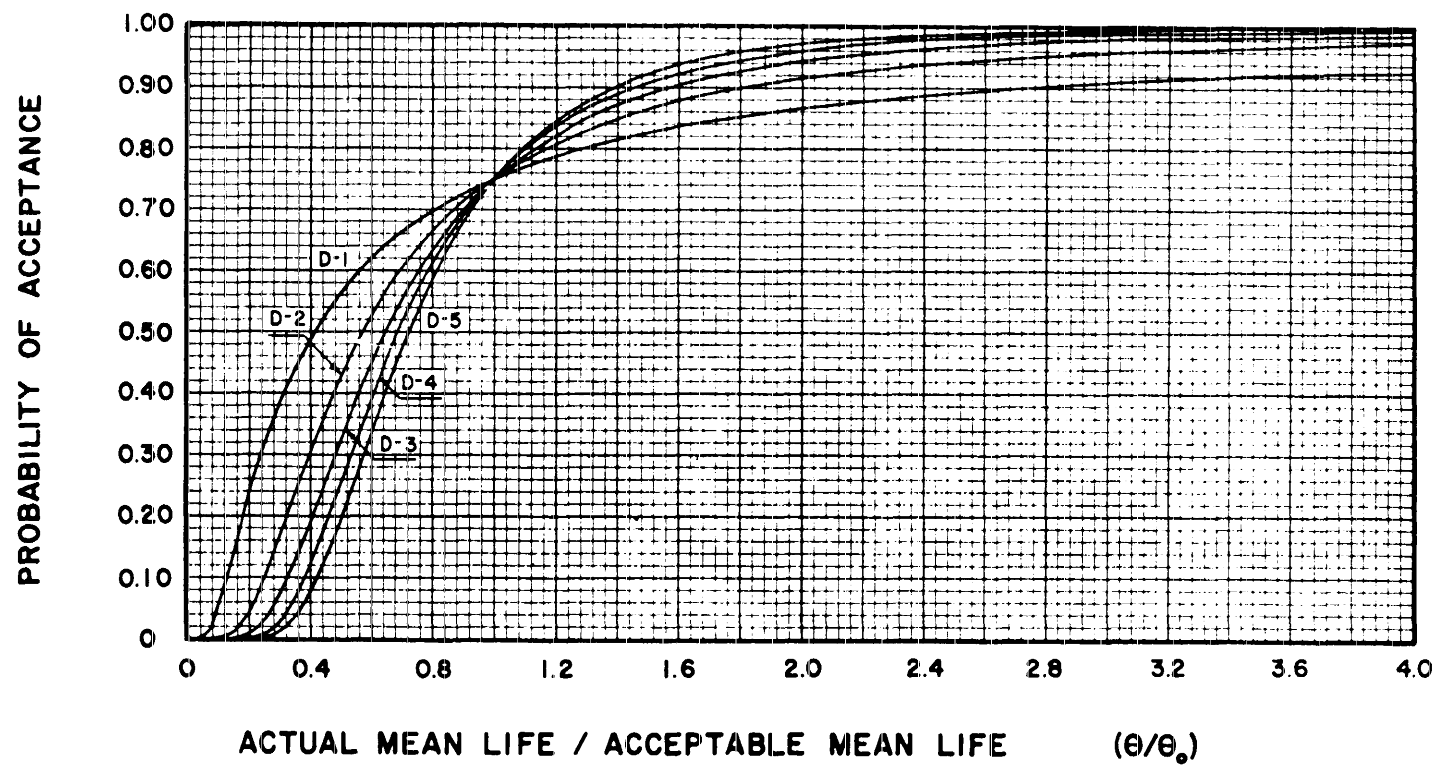

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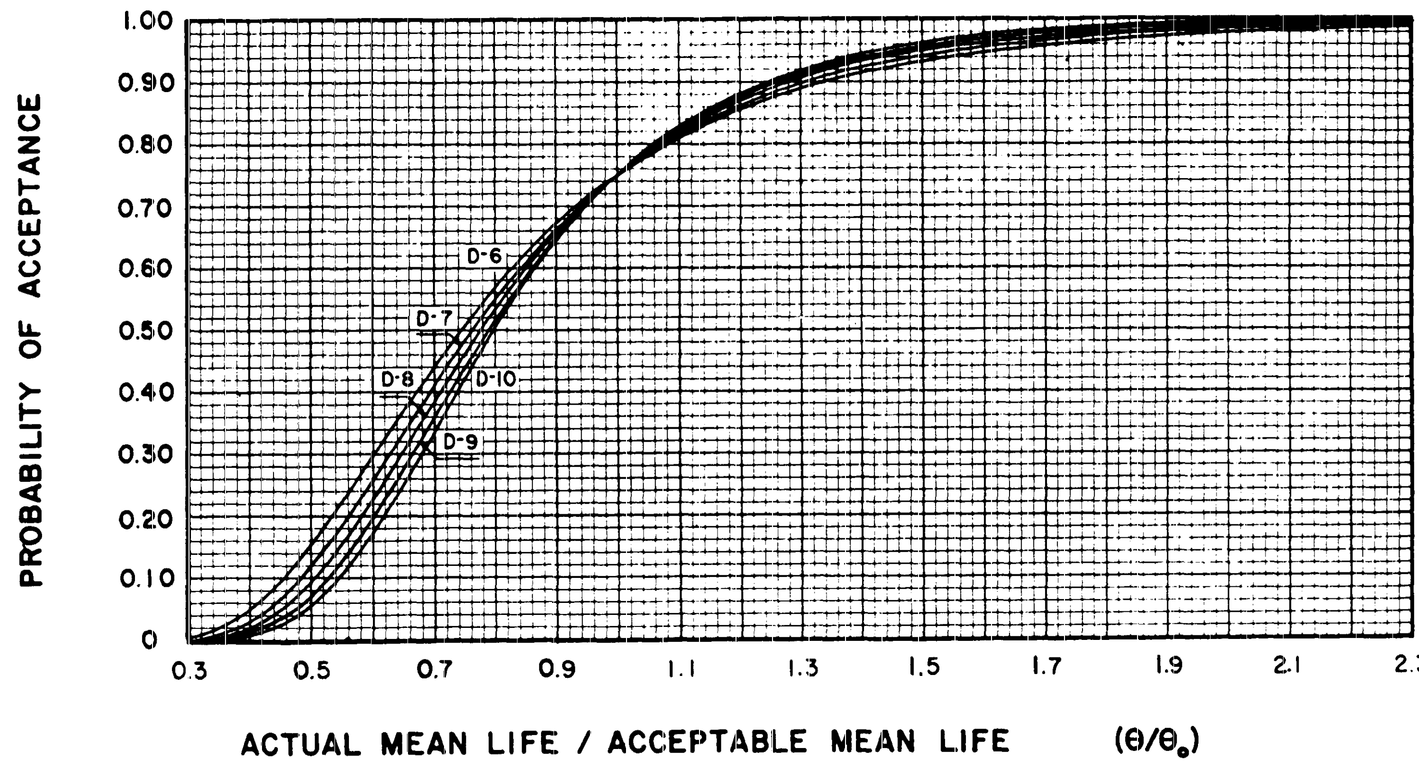

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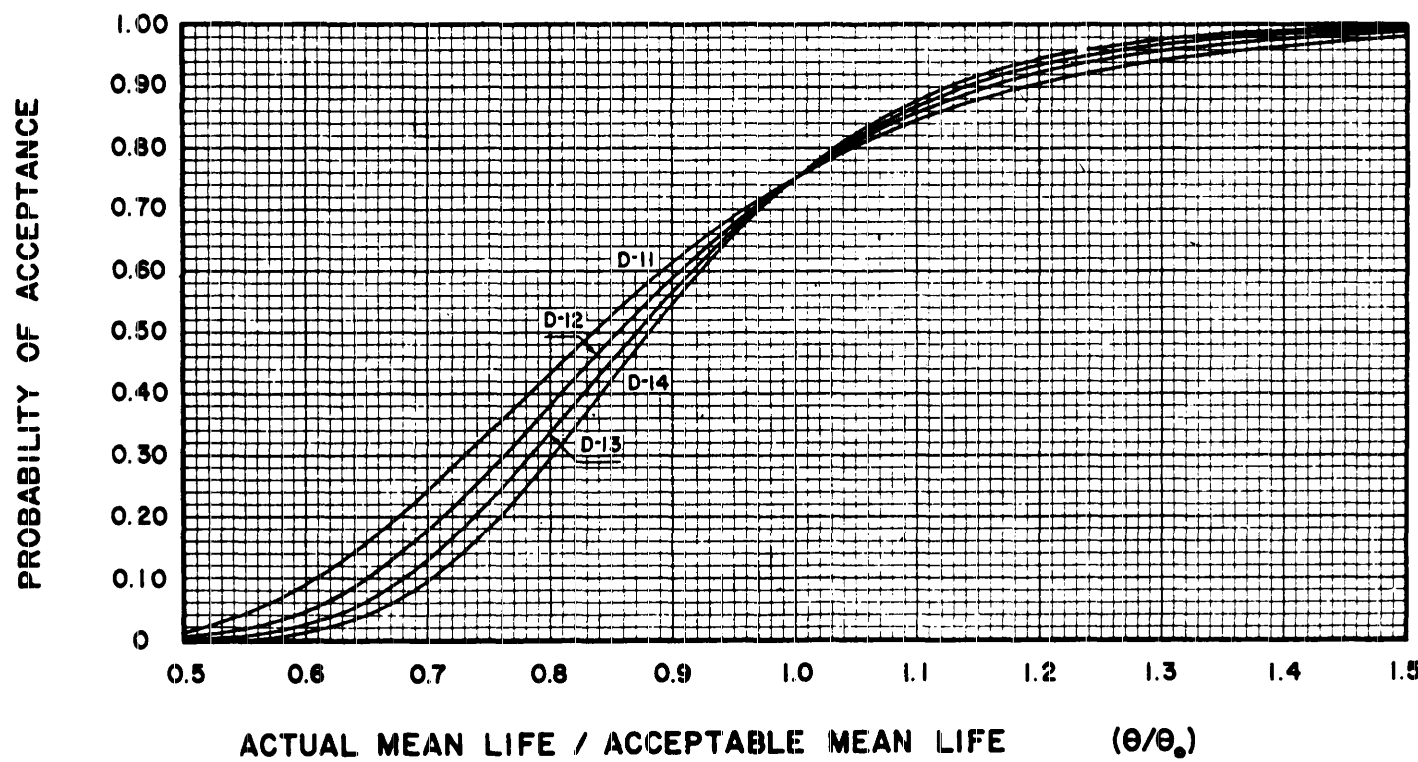

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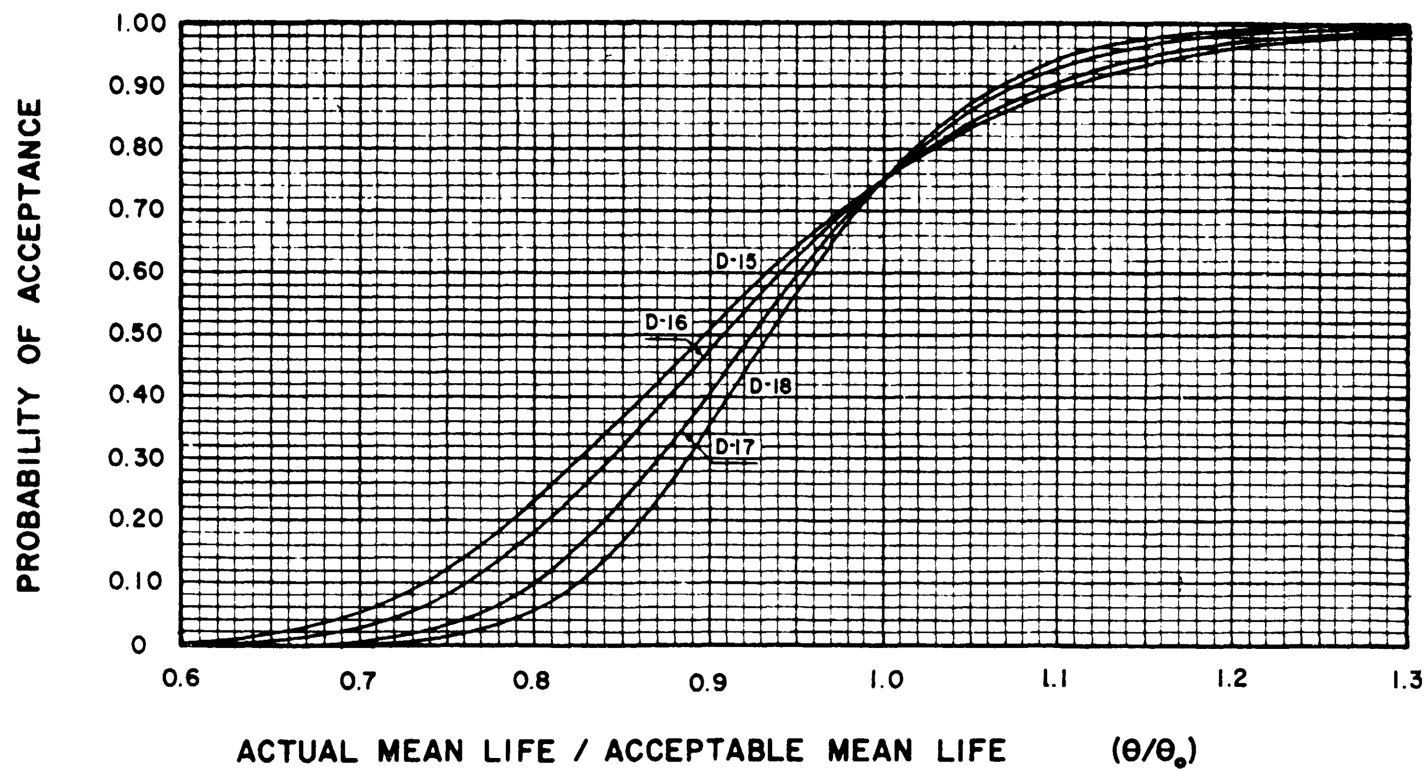

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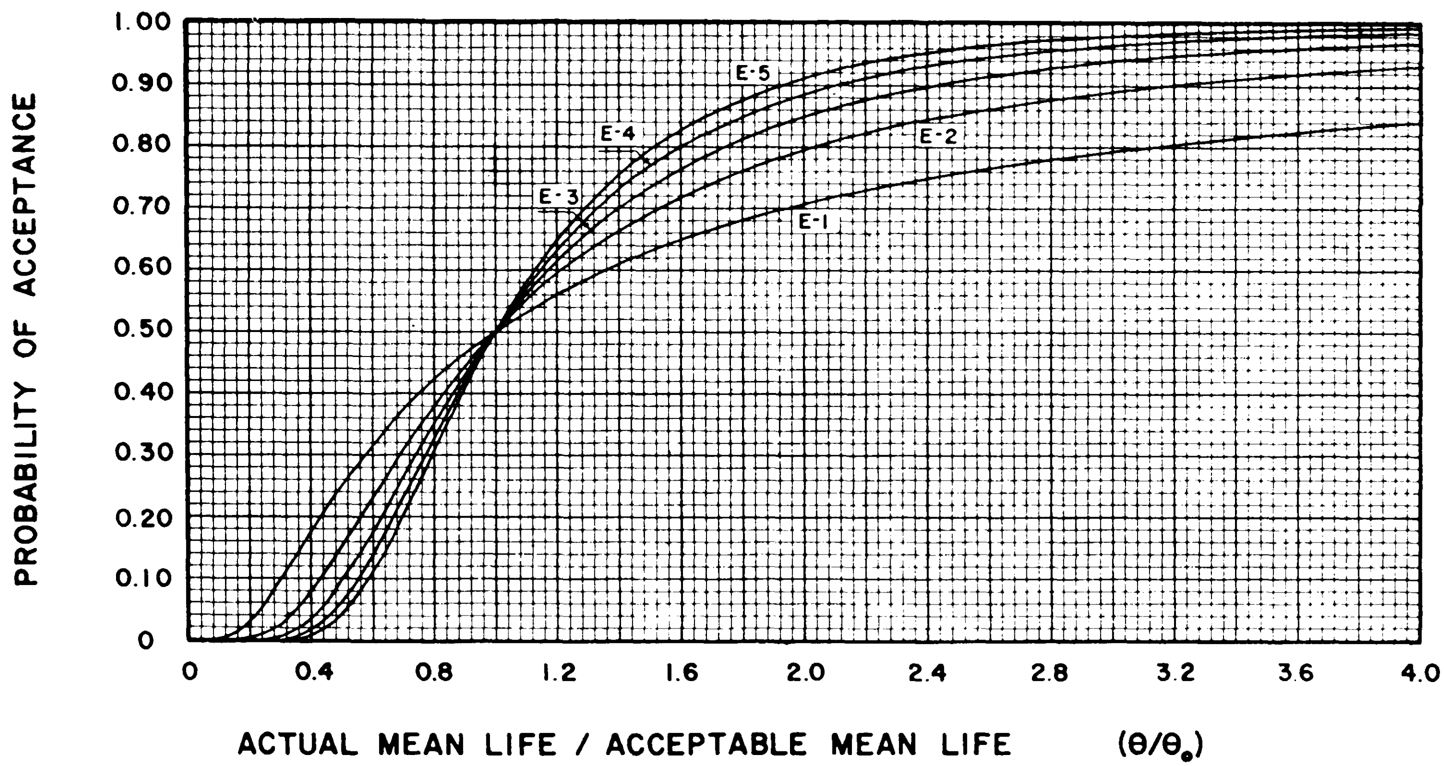

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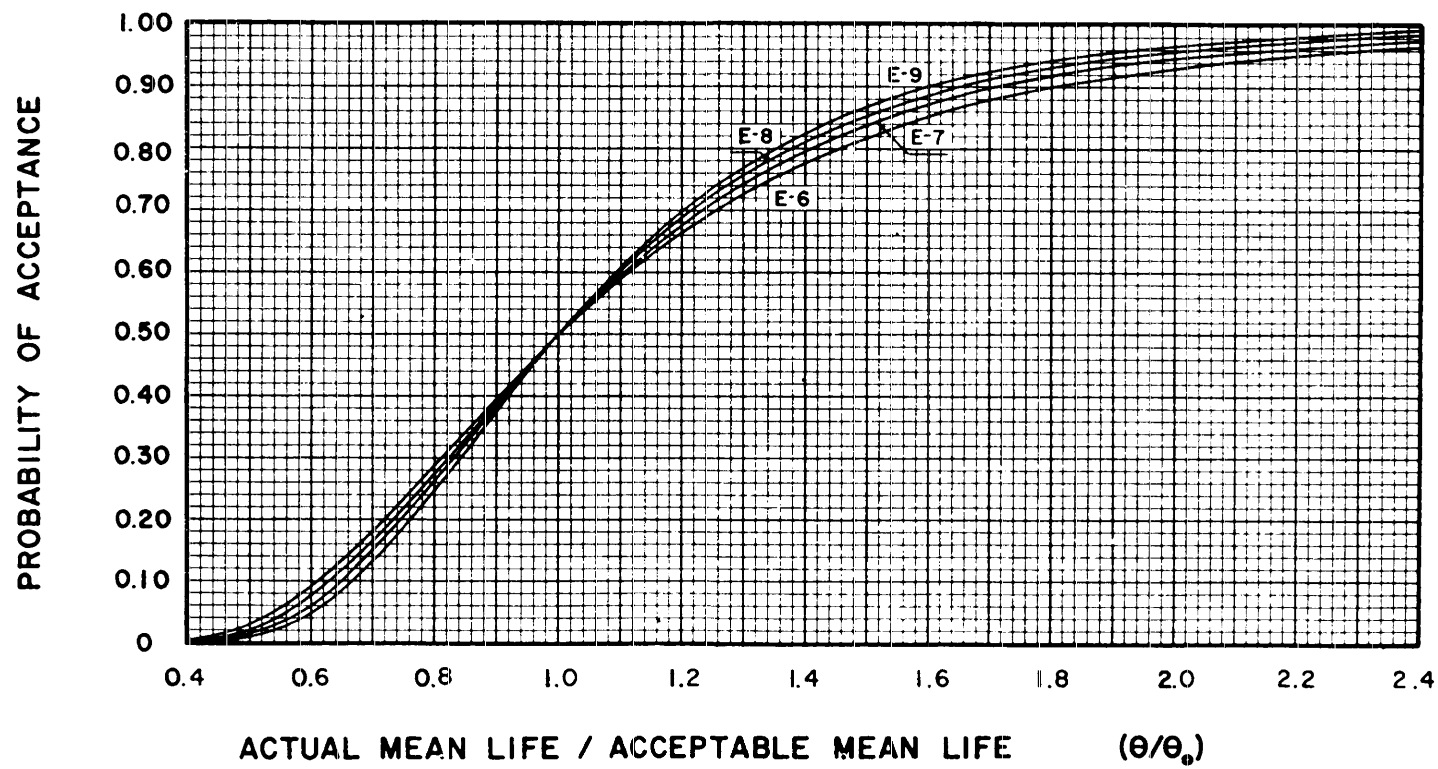

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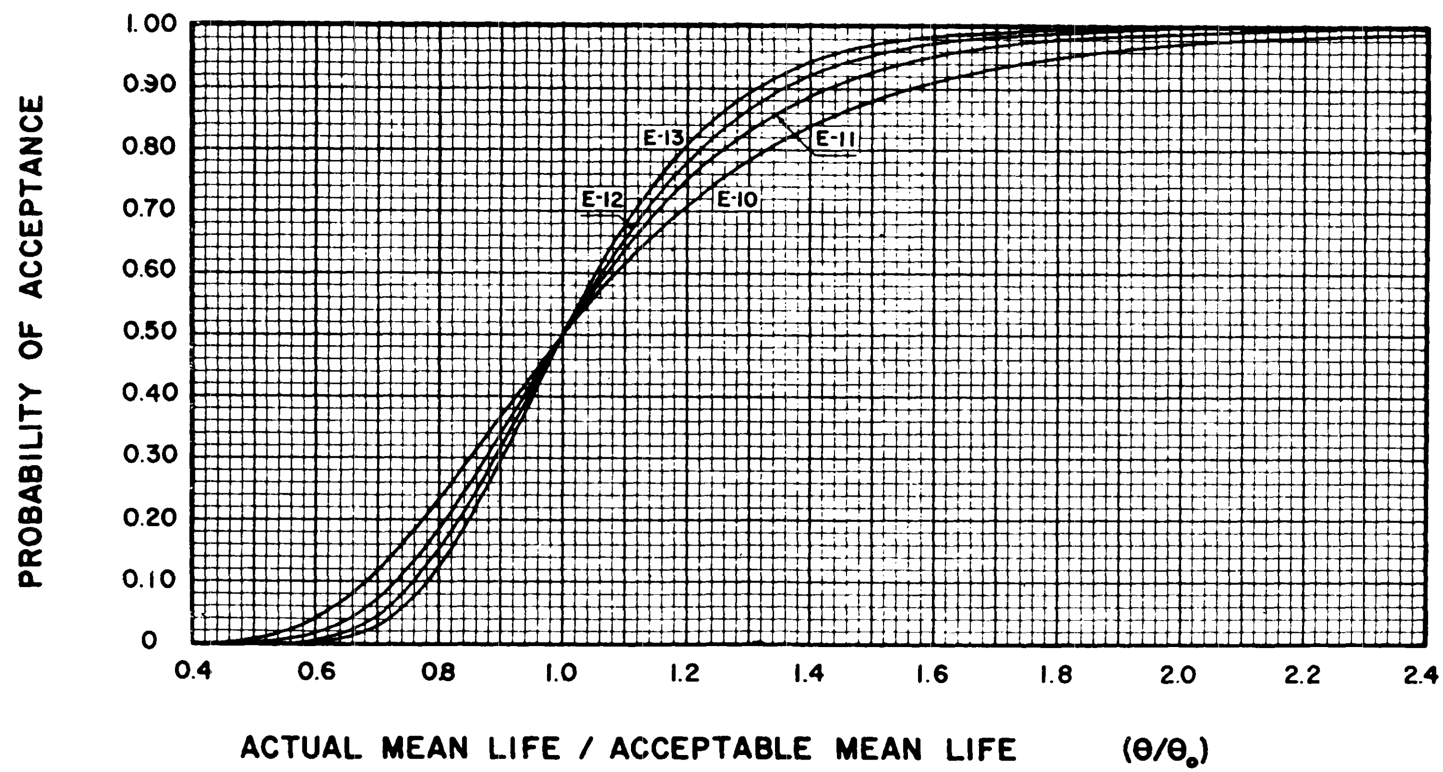

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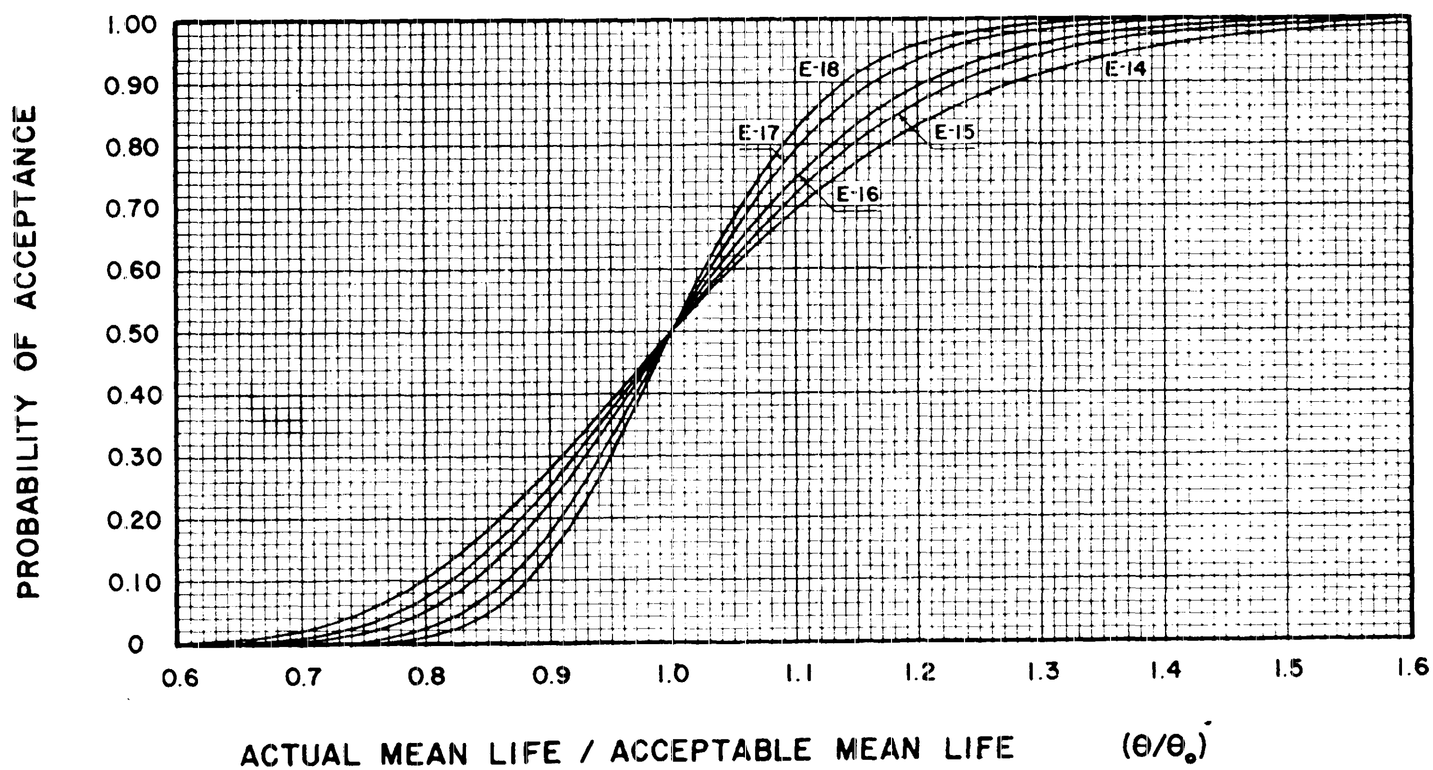

These curves are valid only when sampling from an exponential distribution.

Note: Notations on curves are sample plan code designation

2A4 SPECIFYING ACCEPTANCE PROCEDURES

In order to completely identify the sampling plan to be used, the following must be specified for the sampling plans of:

section 2B, part I .... α, r, θ_0 or the Sample plan code, θ_0

section 2B, part III .... α, β, θ_0, θ_1

for secton 2C, part I .... θ_0, r, α, n or Sample plan code, n, θ_0

for section 2C, part II .... α, β, θ_0, θ_1, T

for section 2C, part III .... α, β, p_1, p_0, T or α, β, G_1, G_0, T

for section 2D .... Sample plan code, θ_0

In addition, the use of life testing with or without replacement may be specified, except when using the sampling plans of section 2C, part III.

SECTION 2B

LIFE TESTS TERMINATED UPON OCCURRENCE OF PREASSIGNED NUMBER OF FAILURES

Part I

ACCEPTANCE PROCEDURES

2B1 LIFE TEST SAMPLING PLANS

This part of the handbook describes the procedures for use with life tests that are terminated upon the occurrence of a preassigned number of failures. Two procedures are given: (1) a procedure when testing without replacement, and (2) another procedure when testing with replacement.

2B1.1 Use of Life Test Sampling Plans. To determine whether the lot meets the acceptability criterion with respect to average length of life, the applicable sampling plan shall be used in accordance with the provisions of section 2A, General Description of Life Test Plans, and those in this part of the handbook.

2B1.2 Drawing of Samples. All samples shall be drawn in accordance with paragraph 1A6.

2B2 SELECTING THE LIFE TEST SAMPLING PLAN

2B2.1 Master Sampling Table. The master sampling table for the life test sampling plans of this part of the handbook is table 2B-1.

2B2.2 Obtaining the Sampling Plan. The life test sampling plan consists of a sample size, a termination number, and an associated acceptability constant. The sampling plan is obtained from master table 2B–1.

2B2.2.1 Sample Sizes. For the procedures of section 2B, part I, the acceptability constants and the OC curves do not depend on the number of units of product placed on test. The sample size, as mentioned in paragraph 1A6.4, depends on the relative cost of placing large numbers of units of product on test and on the expected length of time the life test must continue. The sample size may be selected by using the procedures of section 2B, part II.

2B2.2.2 Termination Number. The termination number, r, may be selected from among those given in table 2B–1 and specified prior to the initiation of the life test. The choice of this number shall be dependent on the degree of protection desired against acceptance of material with unacceptable mean life. The larger the termination number, the larger is the ratio θ_1/θ_0 and, as mentioned in paragraph 2A3.3, the greater is the assurance against accepting material with an unacceptable mean life.

2B2.2.3 Acceptability Constant. The acceptability constant C, corresponding to the applicable termination number r and producer's risk α, is obtained from the master table by multiplying the tabled entry by the acceptable mean life θ_0.

2B3 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITHOUT REPLACEMENT

2B3.1 Estimate of Mean Life. The acceptability of a lot, when using a life test from this part of the handbook, shall be judged by the quantity $\hat{\theta}_{r,n}$.

2B3.2 Computation. The following quantity shall be computed from the test results:

$$\hat{\theta}_{r,n}=\frac{1}{r}\left[\sum_{i=1}^{r} x_{i,n}+(n-r)x_{r,n}\right]$$

where $\hat{\theta}_{r,n}$ is the estimate of the lot mean life

r is the termination number

n is the sample size

$x_{i,n}$ is the time when the i'th failure occurs. $i=1, 2, \ldots, r$.

2B3.3 Acceptability Criterion. Compare the quantity $\hat{\theta}_{r,n}$ with the acceptability constant C, mentioned in paragraph 2B2.2.3. If $\hat{\theta}_{r,n}$ is equal to or greater than C, the lot meets the acceptability criterion; if $\hat{\theta}_{r,n}$ is less than C, then the lot does not meet the acceptability criterion.

2B4 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITH REPLACEMENT

2B4.1 Estimate of Mean Life. The acceptability of a lot, when using a life test from this part of the handbook, shall be judged by the quantity $\hat{\theta}_{r,n}$.

2B4.2 Computation. The following quantity shall be computed from the test results:

$$\hat{\theta}_{r,n}=n\ x_{r,n}/r$$

where $\hat{\theta}_{r,n}$ is the estimate of the lot mean life
- r is the termination number
- n is the original sample size
- $x_{r,n}$ is the time when the r'th failure occurs.

2B4.3 Acceptability Criterion. Compare the quantity $\hat{\theta}_{r,n}$ with the acceptability constant C, mentioned in paragraph 2B2.2.3. If $\hat{\theta}_{r,n}$ is equal to or greater than C, the lot meets the acceptability criterion; if $\hat{\theta}_{r,n}$ is less than C, then the lot does not meet the acceptability criterion.

Example 2B–1

Example of use of Table 2B–1

Example: Find a life test plan which is to be stopped on the occurrence of the fifth failure and will accept a lot having an acceptable mean life of 1,000 hours with probability 0.90.

Solution: In the notation of this chapter, $\theta_0=1{,}000$, $\alpha=0.10$, and $r=5$. In the testing without replacement case—

$$\hat{\theta}_{r,n}=\frac{1}{5}[x_{1,n}+x_{2,n}+x_{3,n}+x_{4,n}+x_{5,n}+(n-5)x_{5,n}]$$

and in the replacement case $\hat{\theta}_{r,n}=n\ x_{5,n}/5$. The acceptability criterion is, accept the lot if—

$$\begin{aligned}\hat{\theta}_{5,n}&\geqq C\\&\geqq \theta_0(C/\theta_0)=(1{,}000)(0.487)=487.\end{aligned}$$

The quantity $C/\theta_0=0.487$ is obtained from table 2B–1. In words, place n items on test. Wait until the first five failures occur. Compute $\hat{\theta}_{5,n}$. Accept the lot if $\hat{\theta}_{5,n}\geqq 487$; reject the lot otherwise.

Remark 1: The code designation for the above life test sampling plan is obtained from table 2B–1 as C–5. From table 2A–2, the probability of accepting a lot with mean life of, say, 500 hours may be obtained by finding the ordinate of the OC curve labeled C–5 at the point where the abscissa $\theta/\theta_0=500/1000=0.5$. The probability is seen to be equal to 0.47.

Remark 2: In this example, if the termination number had been selected as 6 instead of 5, the probability of accepting a lot with mean life of 500 hours is obtained from the OC curve labeled C–6. The probability is seen to equal 0.41. This illustrates the remark made in paragraph 2B2.2.2 that the larger the termination number, the higher the probability of rejecting lots with unacceptable mean life.

Example 2B-2

Example of Calculations

Testing Without Replacement

Example: Suppose that in the life test of example 2B-1, 10 units of product had been placed on test. If the failed units were not replaced and the first five failure times were 50, 75, 125, 250, and 300, determine whether the lot met the acceptability criterion.

Solution: In this case

$$\hat{\theta}_{5,10}=\frac{50+75+125+250+300+5(300)}{5}=460.$$

Since $460<487$, the lot did not meet the acceptability criterion.

Example 2B-3

Example of Calculations

Testing With Replacement

Example: Suppose that in the life test of example 2B-1, 10 units of product had been placed on test. If the failed units were replaced immediately and the first five failure times were 56, 128, 176, 276, and 442, determine whether the lot met the acceptability criterion.

Solution: In this case

$$\hat{\theta}_{5,10}=10(442)/5=884$$

Since $884>487$, the lot met the acceptability criterion.

Table 2B-1

Master Table for Life Tests Terminated upon Occurrence of Preassigned Number of Failures

r	Producer's risk (α)									
	0.01		0.05		0.10		0.25		0.50	
	Code	C/θ_0	Code	C/θ_0	Code	C/θ_0	Code	C/θ_0	Code	C/θ_0
1	A-1	0.010	B-1	0.052	C-1	0.106	D-1	0.288	E-1	0.693
2	A-2	.074	B-2	.178	C-2	.266	D-2	.481	E-2	.839
3	A-3	.145	B-3	.272	C-3	.367	D-3	.576	E-3	.891
4	A-4	.206	B-4	.342	C-4	.436	D-4	.634	E-4	.918
5	A-5	.256	B-5	.394	C-5	.487	D-5	.674	E-5	.934
6	A-6	.298	B-6	.436	C-6	.525	D-6	.703	E-6	.945
7	A-7	.333	B-7	.469	C-7	.556	D-7	.726	E-7	.953
8	A-8	.363	B-8	.498	C-8	.582	D-8	.744	E-8	.959
9	A-9	.390	B-9	.522	C-9	.604	D-9	.760	E-9	.963
10	A-10	.413	B-10	.543	C-10	.622	D-10	.773	E-10	.967
15	A-11	.498	B-11	.616	C-11	.687	D-11	.816	E-11	.978
20	A-12	.554	B-12	.663	C-12	.726	D-12	.842	E-12	.983
25	A-13	.594	B-13	.695	C-13	.754	D-13	.859	E-13	.987
30	A-14	.625	B-14	.720	C-14	.774	D-14	.872	E-14	.989
40	A-15	.669	B-15	.755	C-15	.803	D-15	.889	E-15	.992
50	A-16	.701	B-16	.779	C-16	.824	D-16	.901	E-16	.993
75	A-17	.751	B-17	.818	C-17	.855	D-17	.920	E-17	.996
100	A-18	.782	B-18	.841	C-18	.874	D-18	.931	E-18	.997

Producer's risk α is the probability of rejecting lots with mean life θ_0.
Acceptance criterion: Accept lot if $\hat{\theta}_{r,n} \geqq \theta_0(C/\theta_0)$.
For explanation of the code, see par. 2A3.2 and table 2A-1.

LIFE TESTS TERMINATED UPON OCCURRENCE OF PREASSIGNED NUMBER OF FAILURES

Part II

EXPECTED WAITING TIME OF LIFE TESTS AND COST CONSIDERATIONS IN SELECTION OF SAMPLE SIZES

2B5 SAMPLE SIZES

The operating characteristics of the life test sampling plans of section 2B, part I, are independent of the number of units of product placed on test. Thus, all tests based on common values of the termination number r and producer's risk α are equally good and the choice of the sample size n depends only on the relative cost of placing a large number of units of product on test and on the expected waiting time required for decision. For fixed α and r, increasing n will, on one hand, cut the expected waiting time; but will, on the other hand, increase the cost due to placing more units of product on test. This part of the handbook provides procedures for determining the optimum sample size based on considerations of cost.

2B5.1 Expected Waiting Time. The mean life of the lot and, as noted in paragraph 2B5, the size of the sample drawn from the lot affect the expected waiting time required to observe the r'th failure in a sample of size n. The r'th failure is expected to occur more quickly in samples drawn from lots with low values of mean life. The values of the expected waiting time divided by the mean life of the lot, when testing without replacement, are given in tables 2B–2(a) and 2B–2(b). Corresponding values for the testing with replacement situation are not tabled but may be calculated by dividing the termination number r by the sample size n, i.e.,

$$\frac{\text{Expected Waiting Time}}{\text{Mean Life of Lot}}=\frac{r}{n}$$

2B5.2 Relative Saving in Time by Increasing Sample Size When Testing Without Replacement. When testing without replacement, the expected waiting time required to observe the r'th failure in a sample of size $n, (n \geqq r)$, may be obtained from table 2B–2(a) or 2B–2(b) by multiplying the tabled entry by the mean life of the lot. By dividing the expected waiting time when n units of product are placed on test by that when only r units are placed on test, the mean life of the lot cancels out and the ratio

$$\frac{\text{Expected Waiting Time for } r \text{ Failures in Sample of } n}{\text{Expected Waiting Time for } r \text{ Failures in Sample of } r}$$

is a measure of the relative expected saving in time due to placing more units of product on test. A brief table of these ratios is given in table 2B–3.

2B5.3 Relative Saving in Time by Increasing Sample Size When Testing With Replacement. When testing with replacement, the expected waiting time required to observe the r'th failure in a sample of size n is equal to the quantity $r\theta/n$. By dividing the expected waiting time when n units of product are placed on test by that when only r units are placed on test, the mean life of the lot cancels out and the ratio

$$\text{Relative Saving}=r\theta/n\theta=r/n$$

is a measure of the relative expected saving in time due to using larger sample sizes.

2B5.4 Relative Saving in Time by Testing With Replacement as Compared to Testing Without Replacement. When testing with replacement, the expected waiting time required to observe the r'th failure in a sample of size n $(n \geqq r)$ is equal to the quantity $r\theta/n$. When testing without replacement, this expected waiting time may be obtained from table 2B–2(a) or 2B–2(b) by multiplying the tabled entry by the mean life of the lot θ. By dividing these two expected waiting times, the mean life of the lot cancels out and the ratio

$$\frac{\text{Exp. wait. time for } r \text{ failures in sample of } n \text{ when testing with replacement}}{\text{Exp. wait. time for } r \text{ failures in sample of } n \text{ when testing without replacement}}$$

is a measure of the relative expected saving in time due to sampling with replacement. A brief table of these ratios is given in table 2B–4.

Example 2B–4

Saving in Time by Increasing Sample Size When Testing Without Replacement

Example: Compare the average length of time needed to observe the failure of the first 2 out of 5 units of product under test with the average length of time required to observe the failure of 2 out of 2 units when testing without replacement.

Solution: From table 2B–2(a), it is seen that for $r=2$ and $n=2$, the expected waiting time is 1.5000 θ; and that for $r=2$ and $n=5$, the expected waiting time is 0.4500 θ. Thus the relative saving in time by placing 5 units on test is 0.4500 θ/1.500 θ=0.300. This figure may also be obtained directly from table 2B–3. Hence, the average time required when 5 units are placed on test is 30.0 percent of the average time required when only 2 units are used.

Example 2B–5

Saving in Time by Increasing Sample Size When Testing With Replacement

Example: Make the same comparison as in example 2B–4 if the testing had been with replacement.

Solution: For $r=2$ and $n=2$, the expected waiting time is θ and that for $r=2$ and $n=5$ is $r\theta/n=2\theta/5 = 0.4\theta$. Thus the relative saving in time by placing 5 units on test is $0.4\theta/\theta=0.4$. Hence, the average time required when 5 units are placed on test is 40 percent of the average time required when only 2 units are used.

Example 2B–6

Saving in Time by Testing With Replacement

Example: Compare the average length of time needed to observe the failure of the first 5 out of 5 units of product under test when testing with replacement with the average length of time needed when testing without replacement.

Solution: When testing with replacement, for $r=5$ and $n=5$, the expected waiting time is θ. When testing without replacement, table 2B–2(a) or 2B–2(b) shows that the expected waiting time is 2.2833 θ. Thus the relative saving in time by testing with replacement is θ/2.2833 θ=0.438; or the average time required for a decision, by replacing failed units, is 43.8 percent of the average time required when failed units are not replaced. This figure may also be obtained directly from table 2B–4.

2B5.5 Cost Considerations in Choice of Sample Size. Methods for finding the optimum sample size based on considerations of cost are given in this section.

2B5.5.1 Cost When Testing Without Replacement. The total expected cost of any of the life test plans of section 2B, part I, when testing without replacement, is given by—

$$c_1\theta_0\left(\frac{1}{n}+\frac{1}{n-1}+\ldots+\frac{1}{n-r+1}\right)+c_2n$$

where c_1 is the cost of waiting per unit time

c_2 is the cost of placing a unit of product on test

θ_0 is the acceptable mean life

r is the termination number

n is the sample size

2B5.5.2 Optimum Sample Size When Testing Without Replacement. The value of n which minimizes the total cost, as determined by the method of paragraph 2B5.5.1, is the optimum sample size. A general method of obtaining the optimum n is to use table 2B-2(a) or 2B-2(b). The smallest n is chosen such that the difference between the expected waiting time for the r'th failure when that number of units of product are placed on test and that when $n+1$ units are placed on test, is less than the quantity $c_2/c_1\theta_0$.

Example 2B-7

Life Test Terminated Upon Occurrence of Preassigned Number of Failures Testing Without Replacement

Calculation of Costs

Example: Consider the case where $r=10$, $\theta_0=1{,}000$ hours, $c_1=\$1$ per hour and $c_2=\$100$ per unit of product tested. Using the total cost formula, determine the optimum sample size if failed units are not replaced.

Solution: Using the formula of paragraph 2B5.5.1, the costs for various values of n are—

n	*Expected cost due to waiting*	*Cost of units tested*	*Total cost*
10	2929	1000	3929
11	2020	1100	3120
12	1603	1200	2803
13	1346	1300	2646
14	1168	1400	2568
15	1035	1500	2535
16	931	1600	2531
17	847	1700	2547

The optimum sample size is thus $n=16$.

Example 2B-8

Life Test Terminated Upon Occurrence of Preassigned Number of Failures Testing Without Replacement

Obtaining Optimum Sample Size With Expected Waiting Time

Example: Use table 2B-2(a) to determine the optimum sample size for the problem of example 2B-7.

Solution: The quantity $c_2/c_1\theta_0$ is equal to 0.1 and from table 2B-2(a), the expected waiting times are—

n	*Expected waiting time to observe 10th failure*		*Difference*
	in n	*in n+1*	
10	2. 9290	2. 0199	0. 9091
11	2. 0199	1. 6032	0. 4167
12	1. 6032	1. 3468	0. 2564
13	1. 3468	1. 1682	0. 1786
14	1. 1682	1. 0349	0. 1333
15	1. 0349	0. 9307	0. 1042
16	0. 9307	0. 8467	*0. 0840
17	0. 8467	0. 7773	0. 0694

*The optimum sample size is $n=16$, as was seen in example 2B-7, since that is the smallest sample size for which the difference in expected waiting times is less than $c_2/c_1\theta_0$ or 0.1.

2B5.5.3 Cost When Testing With Replacement. The total expected cost of any of the life test plans of section 2B, part I, when testing with replacement, is given by—

$$c_1\theta_0\frac{r}{n}+c_2(n+r-1)$$

where c_1 is the cost of waiting per unit time
c_2 is the cost of placing a unit of product on test
θ_0 is the acceptable mean life
r is the termination number
n is the sample size.

2B.5.4 Optimum Sample Size When Testing With Replacement. The value of n which minimizes the total cost, as determined by the method of paragraph 2B5.5.3, is the optimum sample size. In general, the optimum n for the case of testing with replacement is the integer nearest to

$$\sqrt{\frac{c_1\theta_0 r}{c_2}+\frac{1}{4}}$$

Example 2B-9

Life Test Terminated Upon Occurrence of Preassigned Number of Failures Testing With Replacement

Calculation of Cost

Example: Consider the problem of example 2B-7, i.e., $r=10$, $\theta_0=1{,}000$, $c_1=\$1$, and $c_2=\$100$. Using the total cost formula, determine the optimum sample size if failed units were replaced.

Solution: Using the formula of paragraph 2B5.5.3, the costs for various values of n are—

n	*Expected cost due to waiting*	*Cost of units tested*	*Total cost*
9	1111	1800	2911
10	1000	1900	2900
11	909	2000	2909

The optimum sample size is thus $n=10$.

Example 2B-10

Life Test Terminated Upon Occurrence of Preassigned Number of Failures Testing With Replacement

Obtaining Optimum Sample Size by Formula

Example: Use the method of paragraph 2B5.5.4 to determine the optimum sample size for the problem of example 2B-9.

Solution: The integer nearest to—

$$\sqrt{\frac{1(1000)(10)}{100}+\frac{1}{4}}=10.012$$

is 10. This is the optimum sample size as was seen in example 2B-9.

LIFE TESTS TERMINATED UPON OCCURRENCE OF PREASSIGNED NUMBER OF FAILURES TESTING WITHOUT REPLACEMENT

Table 2B-2(a)

Expected Waiting Time

Values of: $\dfrac{\text{Expected waiting time for } r \text{ failures in a sample of } n}{\text{Mean life of lot}}$

In the table $r=1(1)n$ and $n=1(1)20(5)30(10)100$

r \ n	1	2	3	4	5	6
1	1.0000	0.5000	0.3333	0.2500	0.2000	0.1667
2		1.5000	0.8333	0.5833	0.4500	0.3667
3			1.8333	1.0833	0.7833	0.6167
4				2.0833	1.2833	0.9500
5					2.2833	1.4500
6						2.4500

Table 2B–2(a)—Continued

r \ n	7	8	9	10	11	12
1	0. 1429	0. 1250	0. 1111	0. 1000	0. 0909	0. 0833
2	0. 3095	0. 2679	0. 2361	0. 2111	0. 1909	0. 1742
3	0. 5095	0. 4345	0. 3790	0. 3361	0. 3020	0. 2742
4	0. 7595	0. 6345	0. 5456	0. 4790	0. 4270	0. 3853
5	1. 0929	0. 8845	0. 7456	0. 6456	0. 5699	0. 5104
6	1. 5929	1. 2179	0. 9956	0. 8456	0. 7365	0. 6532
7	2. 5929	1. 7179	1. 3290	1. 0956	0. 9365	0. 8199
8		2. 7179	1. 8290	1. 4290	1. 1865	1. 0199
9			2. 8290	1. 9290	1. 5199	1. 2699
10				2. 9290	2. 0199	1. 6032
11					3. 0199	2. 1032
12						3. 1032

r \ n	13	14	15	16	17	18
1	0. 0769	0. 0714	0. 0667	0. 0625	0. 0588	0. 0556
2	0. 1603	0. 1484	0. 1381	0. 1292	0. 1213	0. 1144
3	0. 2512	0. 2317	0. 2150	0. 2006	0. 1880	0. 1769
4	0. 3512	0. 3226	0. 2984	0. 2775	0. 2594	0. 2435
5	0. 4623	0. 4226	0. 3893	0. 3609	0. 3363	0. 3150
6	0. 5873	0. 5337	0. 4893	0. 4518	0. 4197	0. 3919
7	0. 7301	0. 6587	0. 6004	0. 5518	0. 5106	0. 4752
8	0. 8968	0. 8016	0. 7254	0. 6629	0. 6106	0. 5661
9	1. 0968	0. 9682	0. 8682	0. 7879	0. 7217	0. 6661
10	1. 3468	1. 1682	1. 0349	0. 9307	0. 8467	0. 7773
11	1. 6801	1. 4182	1. 2349	1. 0974	0. 9896	0. 9023
12	2. 1801	1. 7516	1. 4849	1. 2974	1. 1562	1. 0451
13	3. 1801	2. 2516	1. 8182	1. 5474	1. 3562	1. 2118
14		3. 2516	2. 3182	1. 8807	1. 6062	1. 4118
15			3. 3182	2. 3807	1. 9396	1. 6618
16				3. 3807	2. 4396	1. 9951
17					3. 4396	2. 4951
18						3. 4951

$n = 19$

r		r		r	
1	0. 0526	7	0. 4445	14	1. 2644
2	0. 1082	8	0. 5279	15	1. 4644
3	0. 1670	9	0. 6188	16	1. 7144
4	0. 2295	10	0. 7188	17	2. 0477
5	0. 2962	11	0. 8299	18	2. 5477
6	0. 3676	12	0. 9549	19	3. 5477
		13	1. 0977		

Table 2B-2(a)—Continued

$n=20$

	r		r		r		r
1	0. 0500	6	0. 3462	11	0. 7687	16	1. 5144
2	0. 1026	7	0. 4176	12	0. 8799	17	1. 7644
3	0. 1582	8	0. 4945	13	1. 0049	18	2. 0977
4	0. 2170	9	0. 5779	14	1. 1477	19	2. 5977
5	0. 2795	10	0. 6688	15	1. 3144	20	3. 5977

$n=25$

	r		r		r		r
1	0. 0400	8	0. 3764	15	0. 8870	22	1. 9826
2	0. 0817	9	0. 4352	16	0. 9870	23	2. 3160
3	0. 1251	10	0. 4977	17	1. 0981	24	2. 8160
4	0. 1706	11	0. 5644	18	1. 2231	25	3. 8160
5	0. 2182	12	0. 6358	19	1. 3660		
6	0. 2682	13	0. 7127	20	1. 5326		
7	0. 3209	14	0. 7961	21	1. 7326		

$n=30$

	r		r		r
1	0. 0333	11	0. 4472	21	1. 1660
2	0. 0678	12	0. 4999	22	1. 2771
3	0. 1035	13	0. 5554	23	1. 4021
4	0. 1406	14	0. 6143	24	1. 5450
5	0. 1790	15	0. 6768	25	1. 7117
6	0. 2190	16	0. 7434	26	1. 9117
7	0. 2607	17	0. 8149	27	2. 1617
8	0. 3042	18	0. 8918	28	2. 4950
9	0. 3496	19	0. 9751	29	2. 9950
10	0. 3972	20	1. 0660	30	3. 9950

$n=40$

	r		r		r		r
1	0. 0250	11	0. 3169	21	0. 7308	31	1. 4496
2	0. 0506	12	0. 3514	22	0. 7834	32	1. 5607
3	0. 0770	13	0. 3871	23	0. 8390	33	1. 6857
4	0. 1040	14	0. 4241	24	0. 8978	34	1. 8285
5	0. 1318	15	0. 4626	25	0. 9603	35	1. 9952
6	0. 1603	16	0. 5026	26	1. 0270	36	2. 1952
7	0. 1897	17	0. 5443	27	1. 0984	37	2. 4452
8	0. 2200	18	0. 5877	28	1. 1753	38	2. 7785
9	0. 2513	19	0. 6332	29	1. 2587	39	3. 2785
10	0. 2836	20	0. 6808	30	1. 3496	40	4. 2785

Table 2B-2(a)—Continued

$n=50$

	r		r		r		r
1	0.0200	14	0.3246	27	0.7649	40	1.5702
2	0.0404	15	0.3524	28	0.8084	41	1.6702
3	0.0612	16	0.3810	29	0.8538	42	1.7813
4	0.0825	17	0.4104	30	0.9015	43	1.9063
5	0.1043	18	0.4407	31	0.9515	44	2.0492
6	0.1265	19	0.4720	32	1.0041	45	2.2159
7	0.1492	20	0.5042	33	1.0597	46	2.4159
8	0.1725	21	0.5376	34	1.1185	47	2.6659
9	0.1963	22	0.5720	35	1.1810	48	2.9992
10	0.2207	23	0.6077	36	1.2476	49	3.4992
11	0.2457	24	0.6448	37	1.3191	50	4.4992
12	0.2713	25	0.6832	38	1.3960		
13	0.2976	26	0.7232	39	1.4793		

$n=60$

	r		r		r
1	0.0167	21	0.4263	41	1.1321
2	0.0336	22	0.4520	42	1.1848
3	0.0509	23	0.4783	43	1.2403
4	0.0684	24	0.5053	44	1.2991
5	0.0863	25	0.5331	45	1.3616
6	0.1044	26	0.5617	46	1.4283
7	0.1230	27	0.5911	47	1.4997
8	0.1418	28	0.6214	48	1.5767
9	0.1611	29	0.6526	49	1.6600
10	0.1807	30	0.6849	50	1.7509
11	0.2007	31	0.7182	51	1.8509
12	0.2211	32	0.7527	52	1.9620
13	0.2419	33	0.7884	53	2.0870
14	0.2632	34	0.8255	54	2.2299
15	0.2849	35	0.8639	55	2.3965
16	0.3071	36	0.9039	56	2.5965
17	0.3299	37	0.9456	57	2.8465
18	0.3531	38	0.9891	58	3.1799
19	0.3769	39	1.0345	59	3.6799
20	0.4013	40	1.0821	60	4.6799

$n = 70$

r		r		r		r	
1	0. 0143	19	0. 3140	37	0. 7440	55	1. 5146
2	0. 0288	20	0. 3336	38	0. 7743	56	1. 5813
3	0. 0435	21	0. 3536	39	0. 8056	57	1. 6527
4	0. 0584	22	0. 3740	40	0. 8378	58	1. 7296
5	0. 0736	23	0. 3949	41	0. 8712	59	1. 8130
6	0. 0889	24	0. 4161	42	0. 9057	60	1. 9039
7	0. 1046	25	0. 4379	43	0. 9414	61	2. 0039
8	0. 1204	26	0. 4601	44	0. 9784	62	2. 1150
9	0. 1366	27	0. 4828	45	1. 0169	63	2. 2400
10	0. 1530	28	0. 5061	46	1. 0569	64	2. 3828
11	0. 1696	29	0. 5299	47	1. 0985	65	2. 5495
12	0. 1866	30	0. 5543	48	1. 1420	66	2. 7495
13	0. 2038	31	0. 5793	49	1. 1875	67	2. 9995
14	0. 2214	32	0. 6049	50	1. 2351	68	3. 3328
15	0. 2392	33	0. 6313	51	1. 2851	69	3. 8328
16	0. 2574	34	0. 6583	52	1. 3377	70	4. 8328
17	0. 2759	35	0. 6861	53	1. 3933		
18	0. 2948	36	0. 7146	54	1. 4521		

$n = 80$

r		r		r		r	
1	0. 0125	21	0. 3023	41	0. 7119	61	1. 4177
2	0. 0252	22	0. 3192	42	0. 7376	62	1. 4704
3	0. 0380	23	0. 3365	43	0. 7639	63	1. 5259
4	0. 0510	24	0. 3540	44	0. 7909	64	1. 5848
5	0. 0641	25	0. 3719	45	0. 8187	65	1. 6473
6	0. 0775	26	0. 3900	46	0. 8473	66	1. 7139
7	0. 0910	27	0. 4086	47	0. 8767	67	1. 7853
8	0. 1047	28	0. 4274	48	0. 9070	68	1. 8623
9	0. 1185	29	0. 4467	49	0. 9382	69	1. 9456
10	0. 1326	30	0. 4663	50	0. 9705	70	2. 0365
11	0. 1469	31	0. 4863	51	1. 0038	71	2. 1365
12	0. 1614	32	0. 5067	52	1. 0383	72	2. 2476
13	0. 1761	33	0. 5275	53	1. 0740	73	2. 3726
14	0. 1911	34	0. 5488	54	1. 1111	74	2. 5155
15	0. 2062	35	0. 5705	55	1. 1495	75	2. 6821
16	0. 2216	36	0. 5928	56	1. 1895	76	2. 8821
17	0. 2372	37	0. 6155	57	1. 2312	77	3. 1321
18	0. 2531	38	0. 6387	58	1. 2747	78	3. 4655
19	0. 2692	39	0. 6625	59	1. 3201	79	3. 9655
20	0. 2856	40	0. 6869	60	1. 3677	80	4. 9655

Table 2B-2(a)—Continued

$n = 90$

r		r		r		r	
1	0. 0111	24	0. 3081	47	0. 7326	70	1. 4848
2	0. 0223	25	0. 3233	48	0. 7558	71	1. 5348
3	0. 0337	26	0. 3387	49	0. 7796	72	1. 5875
4	0. 0452	27	0. 3543	50	0. 8040	73	1. 6430
5	0. 0568	28	0. 3702	51	0. 8290	74	1. 7018
6	0. 0686	29	0. 3863	52	0. 8547	75	1. 7643
7	0. 0805	30	0. 4027	53	0. 8810	76	1. 8310
8	0. 0926	31	0. 4194	54	0. 9080	77	1. 9024
9	0. 1047	32	0. 4363	55	0. 9358	78	1. 9794
10	0. 1171	33	0. 4536	56	0. 9644	79	2. 0627
11	0. 1296	34	0. 4711	57	0. 9938	80	2. 1536
12	0. 1422	35	0. 4890	58	1. 0241	81	2. 2536
13	0. 1551	36	0. 5071	59	1. 0553	82	2. 3647
14	0. 1681	37	0. 5257	60	1. 0876	83	2. 4897
15	0. 1812	38	0. 5445	61	1. 1209	84	2. 6326
16	0. 1945	39	0. 5637	62	1. 1554	85	2. 7992
17	0. 2081	40	0. 5834	63	1. 1911	86	2. 9992
18	0. 2218	41	0. 6034	64	1. 2282	87	3. 2492
19	0. 2356	42	0. 6238	65	1. 2666	88	3. 5825
20	0. 2497	43	0. 6446	66	1. 3066	89	4. 0826
21	0. 2640	44	0. 6659	67	1. 3483	90	5. 0826
22	0. 2785	45	0. 6876	68	1. 3918		
23	0. 2932	46	0. 7098	69	1. 4372		

$n = 100$

r		r		r		r	
1	0. 0100	26	0. 2994	51	0. 7082	76	1. 4114
2	0. 0201	27	0. 3129	52	0. 7286	77	1. 4531
3	0. 0303	28	0. 3266	53	0. 7494	78	1. 4966
4	0. 0406	29	0. 3405	54	0. 7707	79	1. 5420
5	0. 0510	30	0. 3545	55	0. 7924	80	1. 5896
6	0. 0616	31	0. 3688	56	0. 8147	81	1. 6396
7	0. 0722	32	0. 3833	57	0. 8374	82	1. 6923
8	0. 0829	33	0. 3980	58	0. 8606	83	1. 7478
9	0. 0938	34	0. 4130	59	0. 8844	84	1. 8066
10	0. 1048	35	0. 4281	60	0. 9088	85	1. 8691
11	0. 1159	36	0. 4435	61	0. 9338	86	1. 9358
12	0. 1272	37	0. 4591	62	0. 9595	87	2. 0072
13	0. 1385	38	0. 4750	63	0. 9858	88	2. 0842
14	0. 1500	39	0. 4911	64	1. 0128	89	2. 1675
15	0. 1616	40	0. 5075	65	1. 0406	90	2. 2584
16	0. 1734	41	0. 5242	66	1. 0692	91	2. 3584
17	0. 1853	42	0. 5411	67	1. 0986	92	2. 4695
18	0. 1974	43	0. 5584	68	1. 1289	93	2. 5945
19	0. 2096	44	0. 5759	69	1. 1601	94	2. 7374
20	0. 2219	45	0. 5938	70	1. 1924	95	2. 9040
21	0. 2344	46	0. 6119	71	1. 2257	96	3. 1040
22	0. 2471	47	0. 6305	72	1. 2602	97	3. 3540
23	0. 2599	48	0. 6493	73	1. 2959	98	3. 6874
24	0. 2729	49	0. 6686	74	1. 3330	99	4. 1874
25	0. 2860	50	0. 6882	75	1. 3714	100	5. 1874

LIFE TESTS TERMINATED UPON OCCURRENCE OF PREASSIGNED NUMBER OF FAILURES—TESTING WITHOUT REPLACEMENT

Table 2B-2(b)

Expected Waiting Time

$$\text{Values of: } \frac{\text{Expected waiting time for } r \text{ failures in a sample of } n}{\text{Mean life of lot}}$$

In the table $r=1(1)10(5)30(10)50(25)100$ and $n=kr$, with $k=1(1)10(10)20$

r \ n	r	$2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$	$20r$
1	1. 0	0. 5000	0. 3333	0. 2500	0. 2000	0. 1667	0. 1429	0. 1250	0. 1111	0. 1000	0. 0500
2	1. 5000	. 5833	. 3667	. 2679	. 2111	. 1742	. 1484	. 1292	. 1144	. 1026	. 0506
3	1. 8333	. 6167	. 3790	. 2742	. 2150	. 1769	. 1503	. 1306	. 1155	. 1035	. 0509
4	2. 0833	. 6345	. 3854	. 2775	. 2170	. 1782	. 1512	. 1313	. 1161	. 1040	. 0510
5	2. 2833	. 6456	. 3893	. 2795	. 2182	. 1790	. 1518	. 1318	. 1164	. 1043	. 0510
6	2. 4500	. 6532	. 3919	. 2809	. 2190	. 1796	. 1522	. 1321	. 1166	. 1044	. 0511
7	2. 5929	. 6587	. 3938	. 2818	. 2196	. 1800	. 1525	. 1323	. 1168	. 1046	. 0511
8	2. 7179	. 6629	. 3952	. 2825	. 2200	. 1803	. 1527	. 1324	. 1169	. 1047	. 0511
9	2. 8290	. 6661	. 3963	. 2831	. 2204	. 1805	. 1528	. 1325	. 1170	. 1047	. 0511
10	2. 9290	. 6688	. 3972	. 2836	. 2207	. 1807	. 1530	. 1326	. 1171	. 1048	. 0512
15	3. 3182	. 6768	. 4000	. 2849	. 2215	. 1812	. 1534	. 1329	. 1173	. 1050	. 0512
20	3. 5977	. 6808	. 4013	. 2856	. 2219	. 1815	. 1536	. 1331	. 1174	. 1051	. 0512
25	3. 8160	. 6832	. 4022	. 2860	. 2221	. 1817	. 1537	. 1332	. 1775	. 1051	. 0512
30	3. 9950	. 6849	. 4027	. 2863	. 2223	. 1818	. 1538	. 1332	. 1176	. 1052	. 0512
40	4. 2785	. 6869	. 4034	. 2866	. 2225	. 1819	. 1539	. 1333	. 1176	. 1052	. 0513
50	4. 4992	. 6882	. 4038	. 2869	. 2226	. 1820	. 1539	. 1334	. 1176	. 1052	. 0513
75	4. 9014	. 6898	. 4044	. 2871	. 2228	. 1821	. 1540	. 1334	. 1177	. 1053	. 0513
100	5. 1874	. 6907	. 4046	. 2873	. 2229	. 1822	. 1540	. 1334	. 1177	. 1053	. 0513

Table 2B-3

Expected Relative Saving in Time by Increasing Sample Size

r \ n	1	2	3	4	5	10	15	20
1	1	0. 50	0. 33	0. 25	0. 20	0. 10	0. 067	0. 050
2		1	. 56	. 39	. 30	. 14	. 092	. 068
3			1	. 59	. 43	. 18	. 12	. 087
4				1	. 62	. 23	. 14	. 104
5					1	. 28	. 17	. 125
10						1	. 35	. 23

The values tabled here are the ratios

$$\frac{\text{Expected waiting time for } r \text{ failures in sample of } n}{\text{Expected waiting time for } r \text{ failures in sample of } r}.$$

For any given value of r, the smaller the tabled value, the greater is the relative expected saving in time. Thus increasing the sample size will decrease the expected waiting time required for decision.

Table 2B-4

Expected Relative Saving in Time by Testing with Replacement

r	Sample size							
	1	2	3	4	5	10	15	20
1	1	1	1	1	1	1	1	1
2		.67	.80	.86	.89	.95	.97	.97
3			.55	.69	.77	.89	.93	.95
4				.48	.62	.84	.89	.92
5					.44	.77	.86	.89
10						.34	.64	.75

The values tabled here are the ratios

$$\frac{\text{Exp. wait. time for } r \text{ failures in sample of } n \text{ when testing with replacement}}{\text{Exp. wait. time for } r \text{ failures in sample of } n \text{ when testing without replacement}}$$

For any given value of r, the smaller the tabled value, the greater is the relative expected saving in time. Thus, as the sample size increases, replacing failed units result in decreasing relative saving in time.

It should be noted that the time saved by replacing failures is at a cost of having placed more units of product on test.

LIFE TESTS TERMINATED UPON OCCURRENCE OF PREASSIGNED NUMBER OF FAILURES

Part III

LIFE TEST SAMPLING PLANS FOR CERTAIN SPECIFIED VALUES OF α, β, AND θ_1/θ_0

2B6 DESIGNING LIFE TEST SAMPLING PLANS

A life test sampling plan may be designed so that its OC curve meets the following prescribed conditions: (1) if $\theta=\theta_0$, then the probability of the lot meeting the acceptability criterion is $1-\alpha$, and (2) if $\theta=\theta_1$, then the probability of the lot meeting the acceptability criterion is less than or equal to β. This part of the handbook, which may be considered an extension of section 2B, part I, provides procedures for obtaining values of the termination number r and the acceptability constant C when certain selected values of α, β, and θ_1/θ_0 are specified. When other values of α, β, and θ_1/θ_0 than those provided in this part of the handbook are needed, refer to section 2B, part I to determine whether one of the life test sampling plans given in that section is applicable.

2B6.1 Life Test Sampling Plans. From table 2B-5, values of the termination number r and the acceptability constant C may be obtained for values of $\alpha=0.01$, 0.05, 0.10, and 0.25; $\beta=0.01$, 0.05, 0.10, and 0.25; and $\theta_1/\theta_0=2/3$, 1/2, 1/3, 1/5, and 1/10. The value of r is obtained directly from table 2B-5 but the acceptability constant C is obtained by multiplying the tabled entry by the acceptable mean life θ_0.

Example 2B-11

Life Tests Terminated Upon Occurrence of Preassigned Number of Failures

Example: Find a life test sampling plan which possesses the following OC curve: If the mean life is $\theta_0=900$ hours, the lot is accepted with probability 0.95; if the mean life is $\theta_1=300$ hours, it is accepted with probability approximately equal to 0.10.

Solution: In this example, $\theta_1/\theta_0=1/3$, $\alpha=0.05$, and $\beta=0.10$. Looking in table 2B-5, the termination number $r=8$ and acceptability constant $C=\theta_0(C/\theta_0)=900(0.4976)=448$ are obtained. In words: Place 8 or more units of product on test. Stop life testing after 8 failures have occurred. If the estimate of lot mean life $\hat{\theta}_{8,n}$ is greater than or equal to 448, the lot is acceptable; otherwise, the lot is not acceptable.

2B6.2 Expansion of Table 2B-5 for Values of θ_1/θ_0 Greater Than 2/3. Approximate values of the termination number r and the acceptability constant C may be obtained to supplement those given in table 2B-5 for values of θ_1/θ_0 greater than 2/3 provided the same values of α and β as given in Table 2B-5 are specified. Compute:

$$r=\left(\frac{K_\beta+(\theta_0/\theta_1)\,K_\alpha}{(\theta_0/\theta_1)-1}\right)^2$$

and

$$C=\theta_0\left(1-\frac{K_\alpha}{\sqrt{r}}\right)$$

where values of K_α and K_β are tabled below.

α or β	K_α or K_β*
0. 01	2. 326
0. 05	1. 645
0. 10	1. 282
0. 25	0. 674

Example 2B-12

Life Tests Terminated Upon Occurrence of Preassigned Number of Failures

Example: Find the appropriate values of the termination number r and the acceptability constant C for the case where the acceptable mean life $\theta_0=110$ hours, the unacceptable mean life $\theta_1=100$ hours, the producer's risk $\alpha=0.05$, and the consumer's risk $\beta=0.10$.

Solution: From the formulas of paragraph 2B6.2

$$r=\left(\frac{1.282+(1.1)\,(1.645)}{0.1}\right)^2=956$$

and

$$C=110\left(1-\frac{1.645}{\sqrt{956}}\right)=104.15.$$

*Values obtained from tables of the cumulative Normal Distribution.

Table 2B-5

Life Test Sampling Plans for Specified α, β, and θ_1/θ_0

θ_1/θ_0	$\alpha=.01$ $\beta=.01$		$\alpha=.01$ $\beta=.05$		$\alpha=.01$ $\beta=.10$		$\alpha=.01$ $\beta=.25$	
	r	C/θ_0	r	C/θ_0	r	C/θ_0	r	C/θ_0
2/3	136	.811	101	.783	83	.762	60	.724
1/2	46	.689	35	.649	30*	.625	22	.572
1/3	19	.544	15*	.498	13	.469	10*	.413
1/5	9*	.390	8*	.363	7*	.333	5*	.256
1/10	5*	.256	4*	.206	4*	.206	3*	.145

θ_1/θ_0	$\alpha=.05$ $\beta=.01$		$\alpha=.05$ $\beta=.05$		$\alpha=.05$ $\beta=.10$		$\alpha=.05$ $\beta=.25$	
	r	C/θ_0	r	C/θ_0	r	C/θ_0	r	C/θ_0
2/3	95	.837	67	.808	55	.789	35	.739
1/2	33	.732	23	.683	19	.655	13	.592
1/3	13	.592	10*	.543	8*	.498	6*	.436
1/5	7*	.469	5*	.394	4*	.342	3*	.272
1/10	4*	.342	3*	.272	3*	.272	2*	.178

θ_1/θ_0	$\alpha=.10$ $\beta=.01$		$\alpha=.10$ $\beta=.05$		$\alpha=.10$ $\beta=.10$		$\alpha=.10$ $\beta=.25$	
	r	C/θ_0	r	C/θ_0	r	C/θ_0	r	C/θ_0
2/3	77	.857	52	.827	41	.806	25*	.754
1/2	26	.758	18	.712	15*	.687	9*	.604
1/3	11	.638	8*	.582	6*	.525	4*	.436
1/5	5*	.487	4*	.436	3*	.367	3*	.367
1/10	3*	.367	2*	.266	2*	.266	2*	.266

θ_1/θ_0	$\alpha=.25$ $\beta=.01$		$\alpha=.25$ $\beta=.05$		$\alpha=.25$ $\beta=.10$		$\alpha=.25$ $\beta=.25$	
	r	C/θ_0	r	C/θ_0	r	C/θ_0	r	C/θ_0
2/3	52	.903	32	.876	23	.853	12	.793
1/2	17	.827	11	.784	8*	.744	5*	.674
1/3	7*	.726	5*	.674	4*	.634	2*	.481
1/5	3*	.576	2*	.481	2*	.481	1*	.288
1/10	2*	.481	2*	.481	1*	.288	1*	.288

Note. A complete set of OC curves is not provided for the sampling plans of this table. For those sampling plans marked (*), the appropriate OC curves in table 2A-2 may be used by determining, from table 2B-1, the sample code designation corresponding to the same values of α and r. For the sampling plans that are not marked (*), two points $(1, 1-\alpha)$ and $(\theta_1/\theta_0, \beta)$ on the OC curves are given.

SECTION 2C

LIFE TESTS TERMINATED AT PREASSIGNED TIME

Part I

ACCEPTANCE PROCEDURES

2C1 LIFE TEST SAMPLING PLANS

This part of the handbook describes the procedures for use with life tests that are terminated at a specified time or upon the occurrence of a specified number of failures, if this number is reached before the specified time. Two procedures are given: (1) a procedure when testing without replacement, and (2) another procedure when testing with replacement.

2C1.1 Use of Life Test Sampling Plans. To determine whether the lot meets the acceptability criterion with respect to average length of life, the applicable sampling plan shall be used in accordance with the provisions of section 2A, General Description of Life Test Plans, and those in this part of the handbook.

2C1.2 Drawing of Samples. All Samples shall be drawn in accordance with paragraph 1A6.

2C2 SELECTING THE LIFE TEST PLAN WHEN SAMPLING WITHOUT REPLACEMENT

2C2.1 Master Sampling Table. The master sampling table for the life test sampling plans of this part of the handbook is table 2C–1.

2C2.2 Obtaining the Sampling Plan. The life test sampling plan consists of a termination number r, a sample size n, and an associated termination time T.

2C2.2.1 Termination Number. The termination number r shall be selected from among those given in table 2C–1 and specified prior to the initiation of the life test. The choice of this number shall be dependent on the degree of protection desired against acceptance of material with unacceptable mean life. The larger the termination number, the larger is the ratio θ_1/θ_0 and, as mentioned in paragraph 2A3.3, the greater is the assurance against accepting material with an unacceptable mean life.

2C2.2.2 Sample Size. The choice of the sample size, as explained in paragraph 1A6.4, is dependent on the relative cost of placing a large number of units of product on test and on the expected waiting time required for a decision. The sample size shall be selected, with this factor in mind, from one of the following multiples of the termination number: $2r$, $3r$, $4r$, $5r$, $6r$, $7r$, $8r$, $9r$, $10r$, and $20r$.

2C2.2.3 Termination Time. The termination time T, corresponding to the applicable termination number r, producer's risk α, and sample size n, is obtained from the master table by multiplying the tabled entry by the acceptable mean life θ_0.

2C3 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITHOUT REPLACEMENT

2C3.1 Acceptability Criterion. The acceptability of a lot with respect to a life test from this part of the handbook shall be judged by the time required for the r'th failure to occur in a sample of size n. Compare the time of the occurrence of the r'th failure with the termination time T, mentioned in paragraph 2C2.2.3. If the r'th failure occurs prior to time T, the lot is considered to have failed to meet the acceptability criterion; if the r'th failure still has not occurred by time T, the lot is considered to have met the acceptability criterion.

2C4 SELECTING THE LIFE TEST PLAN WHEN TESTING WITH REPLACEMENT

2C4.1 Master Sampling Table. The master sampling table for the life test sampling plans with replacement is table 2C–2.

2C4.2 Obtaining the Sampling Plan. The truncated life test sampling plan consists of a termination number r, a sample size n, and an associated termination time T.

2C4.2.1 Termination Number. The termination number r shall be selected from among those given in table 2C–2 and specified prior to the initiation of the life test. The choice of this number shall be dependent on the degree of protection desired against acceptance of material with unacceptable mean life. The larger the termination number, the larger is the ratio θ_1/θ_0, and, as mentioned in paragraph 2A3.3, the greater is the assurance against accepting material with an unacceptable mean life.

2C4.2.2 Sample Size. The choice of the sample size, as explained in paragraph 1A6.4, is dependent on the relative cost of placing a large number of units of product on test and on the expected waiting time required for a decision. The sample size shall be selected, with this factor in mind, from one of the following multiples of the termination number: $2r$, $3r$, $4r$, $5r$, $6r$, $7r$, $8r$, $9r$, $10r$, and $20r$.

2C4.2.3 Termination Time. The termination time T, corresponding to the applicable termination number r, producer's risk α, and sample size n, is obtained from the master table by multiplying the tabled entry by the acceptable mean life θ_0.

2C5 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITH REPLACEMENT

2C5.1 Acceptability Criterion. The acceptability of a lot with respect to a life test shall be judged by the time required for the r'th failure to occur in a sample of size n. Compare the time of the occurrence of the r'th failure with the termination time T, mentioned in paragraph 2C4.2.3. If the r'th failure occurs prior to time T, the lot is considered to have failed to meet the acceptability criterion; if the r'th failure still has not occurred by time T, the lot is considered to have met the acceptability criterion.

Example 2C–1

Life Test Terminated at Preassigned Time

Testing Without Replacement

Example: Find a life test sampling plan without replacement which will accept a lot having an acceptable mean life of 1,000 hours with probability 0.90. The experiment is to be stopped on the occurrence of the fifth failure and 10 units of product are to be placed on test.

Solution: In the notation of this handbook, $\theta_0=1{,}000$, $\alpha=0.10$, $r=5$, and $n=10=2r$. From table 2C–1(c), $T=\theta_0(T/\theta_0)=1{,}000(.314)=314$. In words, accept the lot if the 5th failure has not yet occurred by 314 hours and reject the lot if the 5th failure occurs before 314 hours have elapsed.

Remark 1: The code designation of the above life test sampling plan is obtained from table 2C–1(c) as C–5. From table 2A–2, the probability of accepting a lot with mean life of, say, 500 hours may be obtained by finding the ordinate of the OC curve labeled C–5 at the point where the abscissa $\theta/\theta_0=500/1{,}000=0.5$. The probability of acceptance is seen to be equal to 0.47.

Remark 2: In this example, if the termination number had been selected as 6 rather than 5, the probability of accepting a lot with mean life of 500 hours is obtained from the OC curve labeled C–6. The probability is seen to equal 0.41. This illustrates the remark made in paragraph 2C2.2.1 that the larger the termination number, the higher the probability of rejecting lots with unacceptable mean life.

Example 2C–2

Life Test Terminated at Preassigned Time

Testing With Replacement

Example: In the problem of example 2C–1, find the termination time if the failed units of product had been replaced.

Solution: From table 2C–2(c), $T=\theta_0(T/\theta_0)=1{,}000(.243)=243$ hours. In words, accept the lot if the 5th failure has not occurred by 243 hours and reject the lot if the 5th failure occurs before 243 hours have elapsed.

Remark: The termination time, when sampling with replacement in this example is 243 hours as compared to 314 hours when sampling without replacement. This illustrates the fact that the expected waiting time for a decision as to lot acceptability is lessened by testing with replacement.

Table 2C–1(a)

Master Table for Life Tests Terminated at Preassigned Time—Testing Without Replacement

Values of T/θ_0 for $\alpha=0.01$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
A–1	1	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.0005
A–2	2	.043	.027	.020	.016	.013	.011	.010	.008	.008	.004
A–3	3	.089	.055	.040	.031	.026	.022	.019	.017	.015	.007
A–4	4	.131	.079	.057	.045	.037	.031	.027	.024	.021	.011
A–5	5	.165	.100	.072	.056	.046	.039	.034	.030	.027	.013
A–6	6	.195	.117	.084	.065	.054	.045	.039	.035	.031	.015
A–7	7	.219	.131	.094	.073	.060	.051	.044	.039	.035	.017
A–8	8	.241	.143	.103	.080	.065	.055	.048	.042	.038	.019
A–9	9	.260	.155	.110	.086	.070	.060	.052	.046	.041	.020
A–10	10	.276	.164	.117	.091	.075	.063	.055	.048	.043	.021
A–11	15	.337	.199	.142	.110	.090	.076	.066	.058	.052	.025
A–12	20	.377	.222	.158	.123	.101	.085	.074	.065	.058	.028
A–13	25	.406	.239	.170	.132	.108	.091	.079	.070	.062	.030
A–14	30	.428	.252	.179	.139	.114	.096	.083	.074	.066	.032
A–15	40	.460	.270	.192	.149	.122	.103	.089	.079	.070	.034
A–16	50	.482	.283	.201	.156	.128	.108	.094	.082	.074	.036
A–17	75	.518	.304	.216	.167	.137	.116	.100	.088	.079	.039
A–18	100	.540	.316	.225	.174	.142	.120	.104	.092	.082	.040

For explanation of the code, see par. 2A3.2 and table 2A–1.

Table 2C-1(b)

Master Table for Life Tests Terminated at Preassigned Time—Testing Without Replacement

Values of T/θ_o for $\alpha=0.05$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
B-1	1	0. 026	0. 017	0. 013	0. 010	0. 009	0. 007	0. 006	0. 006	0. 005	0. 003
B-2	2	. 104	. 065	. 048	. 038	. 031	. 026	. 023	. 020	. 018	. 009
B-3	3	. 168	. 103	. 075	. 058	. 048	. 041	. 036	. 031	. 028	. 014
B-4	4	. 217	. 132	. 095	. 074	. 061	. 052	. 045	. 040	. 036	. 017
B-5	5	. 254	. 153	. 110	. 086	. 071	. 060	. 052	. 046	. 041	. 020
B-6	6	. 284	. 170	. 122	. 095	. 078	. 066	. 057	. 051	. 045	. 022
B-7	7	. 309	. 185	. 132	. 103	. 084	. 072	. 062	. 055	. 049	. 024
B-8	8	. 330	. 197	. 141	. 110	. 090	. 076	. 066	. 058	. 052	. 025
B-9	9	. 348	. 207	. 148	. 115	. 094	. 080	. 069	. 061	. 055	. 027
B-10	10	. 363	. 216	. 154	. 120	. 098	. 083	. 072	. 064	. 057	. 028
B-11	15	. 417	. 246	. 175	. 136	. 112	. 094	. 082	. 072	. 065	. 032
B-12	20	. 451	. 266	. 189	. 147	. 120	. 102	. 088	. 078	. 070	. 034
B-13	25	. 475	. 280	. 199	. 154	. 126	. 107	. 093	. 082	. 073	. 036
B-14	30	. 493	. 290	. 206	. 160	. 131	. 111	. 096	. 085	. 076	. 037
B-15	40	. 519	. 305	. 216	. 168	. 137	. 116	. 101	. 089	. 079	. 039
B-16	50	. 536	. 315	. 223	. 173	. 142	. 120	. 104	. 092	. 082	. 040
B-17	75	. 564	. 331	. 235	. 182	. 149	. 126	. 109	. 096	. 086	. 042
B-18	100	. 581	. 340	. 242	. 187	. 153	. 130	. 112	. 099	. 089	. 043

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2C-1(c)

Master Table for Life Tests Terminated at Preassigned Time—Testing Without Replacement

Values of T/θ_o for $\alpha=0.10$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
C-1	1	0. 053	0. 035	0. 026	0. 021	0. 018	0. 015	0. 013	0. 012	0. 011	0. 005
C-2	2	. 155	. 098	. 071	. 056	. 046	. 039	. 034	. 030	. 027	. 013
C-3	3	. 226	. 139	. 101	. 079	. 065	. 055	. 048	. 042	. 038	. 019
C-4	4	. 277	. 168	. 121	. 095	. 078	. 066	. 057	. 051	. 045	. 022
C-5	5	. 314	. 189	. 136	. 106	. 087	. 074	. 064	. 057	. 051	. 025
C-6	6	. 343	. 206	. 147	. 115	. 094	. 080	. 069	. 061	. 055	. 027
C-7	7	. 366	. 219	. 157	. 122	. 100	. 085	. 074	. 065	. 058	. 028
C-8	8	. 386	. 230	. 164	. 128	. 105	. 089	. 077	. 068	. 061	. 030
C-9	9	. 402	. 239	. 171	. 133	. 109	. 092	. 080	. 071	. 063	. 031
C-10	10	. 416	. 247	. 176	. 137	. 112	. 095	. 082	. 073	. 065	. 032
C-11	15	. 465	. 275	. 196	. 152	. 124	. 105	. 091	. 081	. 072	. 035
C-12	20	. 494	. 291	. 207	. 161	. 132	. 112	. 097	. 085	. 076	. 037
C-13	25	. 515	. 303	. 216	. 167	. 137	. 116	. 100	. 089	. 079	. 039
C-14	30	. 530	. 312	. 222	. 172	. 141	. 119	. 103	. 091	. 081	. 040
C-15	40	. 552	. 324	. 230	. 179	. 146	. 124	. 107	. 094	. 084	. 041
C-16	50	. 567	. 333	. 236	. 183	. 150	. 127	. 110	. 097	. 087	. 042
C-17	75	. 590	. 346	. 245	. 190	. 156	. 132	. 114	. 101	. 090	. 044
C-18	100	. 604	. 354	. 251	. 195	. 159	. 135	. 117	. 103	. 092	. 045

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2C-1(d)

Master Table for Life Tests Terminated at Preassigned time—Testing Without Replacement

Values of T/θ_o for $\alpha=0.25$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
D-1	1	0.144	0.096	0.072	0.058	0.048	0.041	0.036	0.032	0.029	0.014
D-2	2	.281	.176	.129	.102	.084	.071	.062	.055	.049	.024
D-3	3	.355	.218	.158	.124	.102	.087	.075	.067	.060	.029
D-4	4	.402	.244	.176	.138	.113	.096	.083	.074	.066	.032
D-5	5	.435	.262	.188	.147	.121	.102	.089	.078	.070	.034
D-6	6	.459	.276	.197	.154	.126	.107	.093	.082	.073	.036
D-7	7	.478	.286	.205	.159	.131	.111	.096	.085	.076	.037
D-8	8	.493	.294	.210	.164	.134	.114	.099	.087	.078	.038
D-9	9	.506	.301	.215	.168	.137	.116	.101	.089	.080	.039
D-10	10	.517	.307	.219	.171	.140	.118	.102	.091	.081	.040
D-11	15	.552	.326	.232	.181	.148	.125	.108	.096	.086	.042
D-12	20	.573	.338	.240	.187	.153	.129	.112	.099	.088	.043
D-13	25	.587	.345	.246	.191	.156	.132	.114	.101	.090	.044
D-14	30	.597	.351	.250	.194	.159	.134	.116	.103	.092	.045
D-15	40	.611	.359	.255	.198	.162	.137	.119	.105	.094	.046
D-16	50	.620	.364	.258	.201	.164	.139	.120	.106	.095	.046
D-17	75	.635	.372	.264	.205	.168	.142	.123	.108	.097	.047
D-18	100	.643	.377	.267	.208	.170	.143	.124	.110	.098	.048

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2C-1(e)

Master Table for Life Tests Terminated at Preassigned Time—Testing Without Replacement

Values of T/θ_0 for $\alpha=0.50$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
E-1	1	0.346	0.231	0.173	0.139	0.116	0.099	0.087	0.077	0.069	0.035
E-2	2	.489	.308	.225	.177	.146	.125	.108	.096	.086	.042
E-3	3	.549	.338	.244	.192	.158	.134	.116	.103	.092	.045
E-4	4	.582	.354	.255	.199	.164	.139	.121	.107	.095	.047
E-5	5	.603	.364	.261	.204	.167	.142	.123	.109	.097	.048
E-6	6	617	.370	.265	.207	.170	.144	.125	.110	.099	.048
E-7	7	.628	.375	.269	.209	.172	.145	.126	.111	.100	.049
E-8	8	.636	.379	.271	.211	.173	.146	.127	.112	.100	.049
E-9	9	.641	.382	.273	.212	.174	.147	.128	.113	.101	.049
E-10	10	.647	.384	.274	.213	.175	.148	.128	.113	.101	.050
E-11	15	.662	.391	.279	.217	.177	.150	.130	.115	.103	.050
E-12	20	.669	.394	.281	.218	.178	.151	.131	.115	.103	.050
E-13	25	.674	.397	.282	.219	.179	.152	.131	.116	.104	.051
E-14	30	.677	.398	.283	.220	.180	.152	.132	.116	.104	.051
E-15	40	.681	.400	.284	.221	.180	.153	.132	.117	.104	.051
E-16	50	.683	.401	.285	.221	.181	.153	.132	.117	.104	.051
E-17	75	.687	.403	.286	.222	.181	.153	.133	.117	.105	.051
E-18	100	.689	.403	.286	.222	.182	.154	.133	.117	.105	.051

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2C–2(a)

Master Table for Life Tests Terminated at Preassigned Time—Testing With Replacement

Values of T/θ_0 for $\alpha=0.01$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
A–1	1	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.0005
A–2	2	.037	.025	.019	.015	.012	.011	.009	.008	.007	.004
A–3	3	.073	.048	.036	.029	.024	.021	.108	.016	.015	.007
A–4	4	.103	.069	.051	.041	.034	.029	.026	.023	.021	.010
A–5	5	.128	.085	.064	.051	.043	.037	.032	.028	.026	.013
A–6	6	.149	.099	.074	.060	.050	.043	.037	.033	.030	.015
A–7	7	.166	.111	.083	.067	.055	.048	.042	.037	.033	.017
A–8	8	.182	.121	.091	.073	.061	.052	.045	.040	.036	.018
A–9	9	.195	.130	.097	.078	.065	.056	.049	.043	.039	.019
A–10	10	.207	.138	.103	.083	.069	.059	.052	.046	.041	.021
A–11	15	.249	.166	.125	.100	.083	.071	.062	.055	.050	.025
A–12	20	.277	.185	.139	.111	.092	.079	.069	.062	.055	.028
A–13	25	.297	.198	.149	.119	.099	.085	.074	.066	.059	.030
A–14	30	.312	.208	.156	.125	.104	.089	.078	.069	.062	.031
A–15	40	.335	.223	.167	.134	.112	.096	.084	.074	.067	.033
A–16	50	.350	.234	.175	.140	.117	.100	.088	.078	.070	.035
A–17	75	.376	.250	.188	.150	.125	.107	.094	.083	.075	.038
A–18	100	.391	.261	.196	.156	.130	.112	.098	.087	.078	.039

For explanation of the code, see par. 2A3.2 and table 2A–1

Table 2C–2(b)

Master Table for Life Tests Terminated at Preassigned Time—Testing With Replacement

Values of T/θ_0 for $\alpha=0.05$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
B–1	1	0.026	0.017	0.013	0.010	0.009	0.007	0.006	0.006	0.005	0.003
B–2	2	.089	.059	.044	.036	.030	.025	.022	.020	.018	.009
B–3	3	.136	.091	.068	.055	.045	.039	.034	.030	.027	.014
B–4	4	.171	.114	.085	.068	.057	.049	.043	.038	.034	.017
B–5	5	.197	.131	.099	.079	.066	.056	.049	.044	.039	.020
B–6	6	.218	.145	.109	.087	.073	.062	.054	.048	.044	.022
B–7	7	.235	.156	.117	.094	.078	.067	.059	.052	.047	.023
B–8	8	.249	.166	.124	.100	.083	.071	.062	.055	.050	.025
B–9	9	.261	.174	.130	.104	.087	.075	.065	.058	.052	.026
B–10	10	.271	.181	.136	.109	.090	.078	.068	.060	.054	.027
B–11	15	.308	.205	.154	.123	.103	.088	.077	.068	.062	.031
B–12	20	.331	.221	.166	.133	.110	.095	.083	.074	.066	.033
B–13	25	.348	.232	.174	.139	.116	.099	.087	.077	.070	.035
B–14	30	.360	.240	.180	.144	.120	.103	.090	.080	.072	.036
B–15	40	.377	.252	.189	.151	.126	.108	.094	.084	.075	.038
B–16	50	.390	.260	.195	.156	.130	.111	.097	.087	.078	.039
B–17	75	.409	.273	.204	.164	.136	.117	.102	.091	.082	.041
B–18	100	.421	.280	.210	.168	.140	.120	.105	.093	.084	.042

For explanation of the code, see par. 2A3.2 and table 2A–1.

Table 2C-2(c)

Master Table for Life Tests Terminated at Preassigned Time—Testing With Replacement

Values of T/θ_0 for $\alpha=0.10$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
C-1	1	0. 053	0. 035	0. 026	0. 021	0. 018	0. 015	0. 013	0. 012	0. 011	0. 005
C-2	2	. 133	. 089	. 066	. 053	. 044	. 038	. 033	. 030	. 027	. 013
C-3	3	. 184	. 122	. 092	. 073	. 061	. 052	. 046	. 041	. 037	. 018
C-4	4	. 218	. 145	. 109	. 087	. 073	. 062	. 055	. 048	. 044	. 022
C-5	5	. 243	. 162	. 122	. 097	. 081	. 070	. 061	. 054	. 049	. 024
C-6	6	. 263	. 175	. 131	. 105	. 088	. 075	. 066	. 058	. 053	. 026
C-7	7	. 278	. 185	. 139	. 111	. 093	. 079	. 070	. 062	. 056	. 028
C-8	8	. 291	. 194	. 146	. 116	. 097	. 083	. 073	. 065	. 058	. 029
C-9	9	. 302	. 201	. 151	. 121	. 101	. 086	. 075	. 067	. 060	. 030
C-10	10	. 311	. 207	. 156	. 124	. 104	. 089	. 078	. 069	. 062	. 031
C-11	15	. 343	. 229	. 172	. 137	. 114	. 098	. 086	. 076	. 069	. 034
C-12	20	. 363	. 242	. 182	. 145	. 121	. 104	. 091	. 081	. 073	. 036
C-13	25	. 377	. 251	. 188	. 151	. 126	. 108	. 094	. 084	. 075	. 038
C-14	30	. 387	. 258	. 194	. 155	. 129	. 111	. 097	. 086	. 077	. 039
C-15	40	. 402	. 268	. 201	. 161	. 134	. 115	. 100	. 089	. 080	. 040
C-16	50	. 412	. 275	. 206	. 165	. 137	. 118	. 103	. 092	. 082	. 041
C-17	75	. 428	. 285	. 214	. 171	. 143	. 122	. 107	. 095	. 086	. 043
C-18	100	. 437	. 291	. 219	. 175	. 146	. 125	. 109	. 097	. 087	. 044

For explanation of the code, see par. 2A3. 2 and table 2A-1.

Table 2C-2(d)

Master Table for Life Tests Terminated at Preassigned Time—Testing With Replacement

Values of T/θ_0 for $\alpha=0.25$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
D-1	1	0. 144	0. 096	0. 072	0. 058	0. 048	0. 041	0. 036	0. 032	0. 029	0. 014
D-2	2	. 240	. 160	. 120	. 096	. 080	. 069	. 060	. 053	. 048	. 024
D-3	3	. 288	. 192	. 144	. 115	. 096	. 082	. 072	. 064	. 058	. 029
D-4	4	. 317	. 211	. 158	. 127	. 106	. 091	. 079	. 070	. 063	. 032
D-5	5	. 337	. 225	. 168	. 135	. 112	. 096	. 084	. 075	. 067	. 034
D-6	6	. 352	. 234	. 176	. 141	. 117	. 100	. 088	. 078	. 070	. 035
D-7	7	. 363	. 242	. 182	. 145	. 121	. 104	. 091	. 081	. 073	. 036
D-8	8	. 372	. 248	. 186	. 149	. 124	. 106	. 093	. 083	. 074	. 037
D-9	9	. 380	. 253	. 190	. 152	. 127	. 109	. 095	. 084	. 076	. 038
D-10	10	. 386	. 258	. 193	. 155	. 129	. 110	. 097	. 086	. 077	. 039
D-11	15	. 408	. 272	. 204	. 163	. 136	. 117	. 102	. 091	. 082	. 041
D-12	20	. 421	. 281	. 210	. 168	. 140	. 120	. 105	. 094	. 084	. 042
D-13	25	. 429	. 286	. 215	. 172	. 143	. 123	. 107	. 095	. 086	. 043
D-14	30	. 436	. 291	. 218	. 174	. 145	. 125	. 109	. 097	. 087	. 044
D-15	40	. 445	. 296	. 222	. 178	. 148	. 127	. 111	. 099	. 089	. 044
D-16	50	. 451	. 300	. 225	. 180	. 150	. 129	. 113	. 100	. 090	. 045
D-17	75	. 460	. 307	. 230	. 184	. 153	. 131	. 115	. 102	. 092	. 046
D-18	100	. 465	. 310	. 233	. 186	. 155	. 133	. 116	. 103	. 093	. 047

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2C-2(e)

Master Table for Life Tests Terminated at Preassigned Time—Testing With Replacement

Values of T/θ_0 for $\alpha=0.50$

Code	r	Sample size									
		2r	3r	4r	5r	6r	7r	8r	9r	10r	20r
E-1	1	0. 347	0. 231	0. 173	0. 139	0. 116	0. 099	0. 087	0. 077	0. 069	0. 035
E-2	2	. 420	. 280	. 210	. 168	. 140	. 120	. 105	. 093	. 084	. 042
E-3	3	. 446	. 297	. 223	. 178	. 149	. 127	. 111	. 099	. 089	. 045
E-4	4	. 459	. 306	. 230	. 184	. 153	. 131	. 115	. 102	. 092	. 046
E-5	5	. 467	. 311	. 234	. 187	. 156	. 133	. 117	. 104	. 093	. 047
E-6	6	. 473	. 315	. 236	. 189	. 158	. 135	. 118	. 105	. 095	. 047
E-7	7	. 476	. 318	. 238	. 191	. 159	. 136	. 119	. 106	. 095	. 048
E-8	8	. 479	. 320	. 240	. 192	. 160	. 137	. 120	. 107	. 096	. 048
E-9	9	. 482	. 321	. 241	. 193	. 161	. 138	. 120	. 107	. 096	. 048
E-10	10	. 483	. 322	. 242	. 193	. 161	. 138	. 121	. 107	. 097	. 048
E-11	15	. 489	. 326	. 244	. 196	. 163	. 140	. 122	. 109	. 098	. 049
E-12	20	. 492	. 328	. 246	. 197	. 164	. 140	. 123	. 109	. 098	. 049
E-13	25	. 493	. 329	. 247	. 197	. 164	. 141	. 123	. 110	. 099	. 049
E-14	30	. 494	. 330	. 247	. 198	. 165	. 141	. 124	. 110	. 099	. 049
E-15	40	. 496	. 331	. 248	. 198	. 165	. 142	. 124	. 110	. 099	. 050
E-16	50	. 497	. 331	. 248	. 199	. 166	. 142	. 124	. 110	. 099	. 050
E-17	75	. 498	. 332	. 249	. 199	. 166	. 142	. 124	. 111	. 100	. 050
E-18	100	. 498	. 332	. 249	. 199	. 166	. 142	. 125	. 111	. 100	. 050

For explanation of the code, see par. 2A3.2 and table 2A-1.

LIFE TESTS TERMINATED AT PREASSIGNED TIME

Part II

LIFE TEST SAMPLING PLANS FOR SPECIFIED α, β, θ_1/θ_0, AND T/θ_0

2C6 DESIGNING LIFE TEST SAMPLING PLANS

A life test sampling plan may be designed so that its OC curve meets the following prescribed conditions: (1) if $\theta=\theta_0$, then the probability of the lot meeting the acceptability criterion is $1-\alpha$, and (2) if $\theta=\theta_1$, then the probability of the lot meeting the acceptability criterion is approximately equal to β. This part of the handbook provides procedures for obtaining values of the termination number and the sample size when certain selected values of α, β, θ_1/θ_0, and T/θ_0 are specified. This part of the handbook may be considered an extension of section 2C, part I, so that when values of α, β, θ_1/θ_0, and T/θ_0 other than those provided here are needed, refer to section 2C, part I, to determine whether one of the sampling plans given in that section is applicable. Moreover, if the desired value of T/θ_0 is not given in this part of the handbook, it may be noted that T is usually an upper limit while θ_0 is a lower limit. Thus, if no sampling plan is given for the desired value of T/θ_0, the sampling plan for the next lower value of T/θ_0 which is given, may be used.

2C6.1 Life Test Plans When Testing Without Replacement. From table 2C-3, values of the termination number r and the sample size n may be obtained when testing without replacement for values of $\alpha=0.01$, 0.05, 0.10, and 0.25; $\beta=0.01$, 0.05, 0.10, and 0.25; $\theta_1/\theta_0=2/3$, 1/2, 1/3, 1/5, and 1/10; and $T/\theta_0=$ 1/3, 1/5, 1/10, and 1/20. The values of θ_1/θ_0 and T/θ_0 shall be computed and for the appropriate values of α and β, values of r and n shall then be obtained from table 2C-3.

Example 2C-3

Life Test Terminated at Preassigned Time

Testing Without Replacement

Example: Find a nonreplacement life test sampling plan which is not to exceed 500 hours and which will accept a lot with acceptable mean life of 10,000 hours at least 90 percent of the time but will reject a lot with unacceptable mean life of 2,000 hours about 90 percent of the time.

Solution: In this case $T=500$, $\theta_0=10{,}000$, $\theta_1=2{,}000$, $\alpha=0.10$, and $\beta=0.10$. Hence, $\theta_1/\theta_0=1/5$, and $T/\theta_0=1/20$. Looking in table 2C-3 under $\alpha=0.10$, $\beta=0.10$, $\theta_1/\theta_0=1/5$, and $T/\theta_0=1/20$, the termination number $r=3$ and sample size $n=23$ are obtained. Thus the desired nonreplacement plan is as follows: Start the life test with $n=23$ units of product. Do not replace any units that fail. The lot is considered to have met the acceptability criterion if 3 failures have not occurred by 500 hours and the life test is terminated at this time. The lot is considered to have failed to meet the acceptability criterion if the 3d failure occurs before 500 hours and the test is terminated at the time of the 3d failure.

2C6.2 Life Test Plans When Testing With Replacement. From table 2C-4, values of the termination number r and the sample size n may be obtained when testing with replacement for values of $\alpha=0.01$, 0.05, 0.10, and 0.25; $\beta=0.01$, 0.05, 0.10, and 0.25; $\theta_1/\theta_0=2/3$, 1/2, 1/3, 1/5, and 1/10; and $T/\theta_0=1/3$, 1/5, 1/10, and 1/20. The values θ_1/θ_0 and T/θ_0 shall be computed and for the appropriate values of α and β, values of r and n shall then be obtained from table 2C-4.

Example 2C-4

Life Test Terminated at Preassigned Time

Testing With Replacement

Example: Find a replacement life test sampling plan which is not to exceed 500 hours and which will accept a lot with mean life of 10,000 hours at least 90 percent of the time but will reject a lot with mean life of 2,000 hours about 90 percent of the time.

Solution: In this problem, $T=500$, $\theta_0=10{,}000$, $\theta_1=2{,}000$, $\alpha=0.10$, and $\beta=0.10$. Hence, $\theta_1/\theta_0=1/5$, $T/\theta_0=1/20$, the termination number $r=3$, and the sample size $n=22$ are obtained. Thus the desired test plan is as follows: Start the life test with $n=22$ units of product. As soon as a unit of product fails, replace it with a new unit. The lot is considered to have met the acceptability criterion if 3 failures have not occurred by 500 hours and the life test is terminated at this time. The lot is considered to have failed to meet the acceptability criterion if the 3d failure occurs before 500 hours and the test is terminated at the time of the 3d failure.

Table 2C-3

Life Test Sampling Plans for Specified α, β, θ_1/θ_0, and T/θ_0

θ_1/θ_0	r	T/θ_0 = 1/3	1/5	1/10	1/20	r	T/θ_0 = 1/3	1/5	1/10	1/20
		n	n	n	n		n	n	n	n
		$\alpha=0.01$		$\beta=0.01$			$\alpha=0.05$		$\beta=0.01$	
2/3	136	403	622	1172	2275	95	289	447	843	1639
1/2	46	119	182	340	657	33	90	138	258	499
1/3	19	41	61	113	216	13	30	45	83	160
1/5	9	15	22	39	74	7	13	20	36	69
1/10	5	6	9	15	28	4	6	9	15	29
		$\alpha=0.01$		$\beta=0.05$			$\alpha=0.05$		$\beta=0.05$	
2/3	101	291	448	842	1632	67	198	305	575	1116
1/2	35	87	132	245	472	23	59	90	168	326
1/3	15	30	45	82	157	10	21	32	59	113
1/5	8	13	18	33	62	5	8	12	22	41
1/10	4	4	6	10	18	3	4	5	9	17
		$\alpha=0.01$		$\beta=0.10$			$\alpha=0.05$		$\beta=0.10$	
2/3	83	234	359	675	1307	55	159	245	462	895
1/2	30	72	109	202	390	19	47	72	134	258
1/3	13	25	37	67	128	8	16	24	43	83
1/5	7	11	15	26	50	4	6	9	15	29
1/10	4	4	6	10	18	3	4	5	9	17
		$\alpha=0.01$		$\beta=0.25$			$\alpha=0.05$		$\beta=0.25$	
2/3	60	162	248	465	899	35	96	147	276	535
1/2	22	49	74	137	262	13	30	45	83	160
1/3	10	18	26	46	87	6	11	16	29	55
1/5	5	6	9	15	28	3	4	5	9	17
1/10	3	3	4	6	10	2	2	2	4	8

Table 2C-3—Continued

θ_1/θ_0	r	T/θ_0 1/3 n	1/5 n	1/10 n	1/20 n	r	T/θ_0 1/3 n	1/5 n	1/10 n	1/20 n
		$\alpha=0.10$		$\beta=0.01$			$\alpha=0.25$		$\beta=0.01$	
2/3	77	238	369	699	1358	52	168	261	496	965
1/2	26	73	112	210	407	17	51	79	149	289
1/3	11	27	40	75	145	7	19	29	54	105
1/5	5	10	14	26	51	3	6	10	18	36
1/10	3	5	7	12	23	2	3	5	10	20
		$\alpha=0.10$		$\beta=0.05$			$\alpha=0.25$		$\beta=0.05$	
2/3	52	156	242	456	886	32	101	156	296	576
1/2	18	48	73	137	265	11	31	48	91	177
1/3	8	18	27	50	97	5	12	19	36	69
1/5	4	7	10	19	36	2	3	5	10	20
1/10	2	2	3	6	11	2	3	5	10	20
		$\alpha=0.10$		$\beta=0.10$			$\alpha=0.25$		$\beta=0.10$	
2/3	41	121	186	351	681	23	71	110	207	403
1/2	15	39	59	110	213	8	22	33	63	123
1/3	6	12	18	34	66	4	9	14	27	52
1/5	3	5	7	12	23	2	3	5	10	20
1/10	2	2	3	6	11	1	1	1	3	6
		$\alpha=0.10$		$\beta=0.25$			$\alpha=0.25$		$\beta=0.25$	
2/3	25	69	107	201	389	12	34	53	101	196
1/2	9	21	31	58	113	5	12	19	36	69
1/3	4	7	10	19	36	2	3	5	10	20
1/5	3	5	7	12	23	1	1	1	3	6
1/10	2	2	3	6	11	1	1	1	3	6

No operating characteristic curves are provided for these sampling plans. However, two points on the OC curves $(1, 1-\alpha)$ and $(\theta_1/\theta_0, \beta)$ are given.

Table 2C-4

Life Test Sampling Plans for Specified α, β, θ_1/θ_0, and T/θ_0

θ_1/θ_0	r	T/θ_0				r	T/θ_0			
		1/3	1/5	1/10	1/20		1/3	1/5	1/10	1/20
		n	n	n	n		n	n	n	n
		$\alpha=0.01$		$\beta=0.01$			$\alpha=0.05$		$\beta=0.01$	
2/3	136	331	551	1103	2207	95	238	397	795	1591
1/2	46	95	158	317	634	33	72	120	241	483
1/3	19	31	51	103	206	13	23	38	76	153
1/5	9	10	17	35	70	7	9	16	32	65
1/10	5	4	6	12	25	4	4	6	13	27
		$\alpha=0.01$		$\beta=0.05$			$\alpha=0.05$		$\beta=0.05$	
2/3	101	237	395	790	1581	67	162	270	541	1082
1/2	35	68	113	227	454	23	47	78	157	314
1/3	15	22	37	74	149	10	16	27	54	108
1/5	8	8	14	29	58	5	6	10	19	39
1/10	4	3	4	8	16	3	3	4	8	16
		$\alpha=0.01$		$\beta=0.10$			$\alpha=0.05$		$\beta=0.10$	
2/3	83	189	316	632	1265	55	130	216	433	867
1/2	30	56	93	187	374	19	37	62	124	248
1/3	13	18	30	60	121	8	11	19	39	79
1/5	7	7	11	23	46	4	4	7	13	27
1/10	4	2	4	8	16	3	3	4	8	16
		$\alpha=0.01$		$\beta=0.25$			$\alpha=0.05$		$\beta=0.25$	
2/3	60	130	217	434	869	35	77	129	258	517
1/2	22	37	62	125	251	13	23	38	76	153
1/3	10	12	20	41	82	6	7	13	26	52
1/5	5	4	7	13	25	3	3	4	8	16
1/10	3	2	2	4	8	2	1	2	3	7

Table 2C-4—Continued

θ_1/θ_0	r	T/θ_0 1/3	1/5	1/10	1/20	r	T/θ_0 1/3	1/5	1/10	1/20
		n	n	n	n		n	n	n	n
		$\alpha=0.10$		$\beta=0.01$			$\alpha=0.25$		$\beta=0.01$	
2/3	77	197	329	659	1319	52	140	234	469	939
1/2	26	59	98	197	394	17	42	70	140	281
1/3	11	21	35	70	140	7	15	25	50	101
1/5	5	7	12	24	48	3	5	8	17	34
1/10	3	3	5	11	22	2	2	4	9	19
		$\alpha=0.10$		$\beta=0.05$			$\alpha=0.25$		$\beta=0.05$	
2/3	52	128	214	429	859	32	84	140	280	560
1/2	18	38	64	128	256	11	25	43	86	172
1/3	8	13	23	46	93	5	10	16	33	67
1/5	4	5	8	17	34	2	3	5	10	19
1/10	2	2	3	5	10	2	2	4	9	19
		$\alpha=0.10$		$\beta=0.10$			$\alpha=0.25$		$\beta=0.10$	
2/3	41	99	165	330	660	23	58	98	196	392
1/2	15	30	51	102	205	8	17	29	59	119
1/3	6	9	15	31	63	4	7	12	25	50
1/5	3	4	6	11	22	2	3	4	9	19
1/10	2	2	2	5	10	1	1	2	3	5
		$\alpha=0.10$		$\beta=0.25$			$\alpha=0.25$		$\beta=0.25$	
2/3	25	56	94	188	376	12	28	47	95	190
1/2	9	16	27	54	108	5	10	16	33	67
1/3	4	5	8	17	34	2	2	4	9	19
1/5	3	3	5	11	22	1	1	2	3	6
1/10	2	1	2	5	10	1	1	1	2	5

No operating characteristic curves are provided for these sampling plans. However, two points on the OC curves $(1, 1-\alpha)$ and $(\theta_1/\theta_0, \beta)$ are given.

LIFE TESTS TERMINATED AT PREASSIGNED TIME—TESTING WITHOUT REPLACEMENT

Part III

LIFE TEST SAMPLING PLANS BASED ON FAILURE RATES

2C7 FRACTION OF LOT FAILING BEFORE SPECIFIED TIME

The sampling plans in this part of the handbook may be used when either (1) the proportion of lot failing before specified time, or (2) the failure rate during time period is specified. Since the sampling plans are based on proportion of lot failing, when the failure rate for period of time is specified, the failure rate shall be multiplied by the specified length of time. That is,

$$p = GT$$

where p is the proportion of lot failing before specified time T and G is the failure rate during period of time T.

2C8 LIFE TEST SAMPLING PLAN

A lot may be considered satisfactory if the failure rate is less than or equal to p_0 and it may be considered unsatisfactory if the failure rate is greater than or equal to p_1, where $p_1 > p_0$. From table 2C-5, values of the termination number r and the factor D are obtained when testing without replacement for values of $\alpha=0.01$, 0.05, and 0.10; $\beta=0.01$, 0.05, and 0.10; and $p_1/p_0=1.5$, 2, 2.5, 3, 4, 5, and 10. In this table the sample size, n, is given by $[D/p_0]$ which means the greatest integer less than or equal to D/p_0, e.g., $[3.8]=3$ and $[4]=4$.

Example 2C-5

Life Test Terminated at Preassigned Time

Testing Without Replacement

Example: Find a nonreplacement life test sampling plan which will accept at least 90 percent of the lots for which the failure rate for period of time, expressed as a percentage, is less than or equal to 1 percent per 1,000 hours and will reject at least 95 percent of the lots for which the failure rate is greater than or equal to 10 percent per 1,000 hours.

Solution: In this problem, $G_0=0.01/1{,}000=0.00001$, $G_1=0.10/1{,}000=0.00010$, $\alpha=0.10$, $\beta=0.05$, and $T=1{,}000$ hours. Thus, $p_0=1{,}000\ G_0=0.01$, $p_1=1{,}000\ G_1=0.10$, and $p_1/p_0=10$. Looking in table 2C-5, it is seen that $r=2$ and $n=[0.532/.01]=53$. Thus, the desired plan is as follows: Place 53 units of product on test. If 2 failures occur before time T, reject the lot and terminate the test at the time of occurrence of the second failure. If one or fewer failures have occurred at time T, accept the lot and terminate the test.

LIFE TESTS TERMINATED AT PREASSIGNED TIME
TESTING WITHOUT REPLACEMENT

Table 2C-5

Life Test Sampling Plans for Specified α, β, and p_1/p_0

Values of r (upper numbers) and of D (lower numbers)*

p_1/p_0	$\alpha=0.01$			$\alpha=0.05$			$\alpha=0.10$		
	$\beta=0.01$	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
3/2	136	101	83	95	67	55	77	52	41
	110.4	79.1	63.3	79.6	54.1	43.4	66.0	43.0	33.0
2	46	35	30	33	23	19	26	18	15
	31.7	22.7	18.7	24.2	15.7	12.4	19.7	12.8	10.3
5/2	27	21	18	19	14	11	15	11	9
	16.4	11.8	9.62	12.4	8.46	6.17	10.3	7.02	5.43
3	19	15	13	13	10	8	11	8	6
	10.3	7.48	6.10	7.69	5.43	3.98	7.02	4.66	3.15
4	12	10	9	9	7	6	7	5	4
	5.43	4.13	3.51	4.70	3.29	2.61	3.90	2.43	1.75
5	9	8	7	7	5	4	5	4	3
	3.51	2.91	2.33	3.29	1.97	1.37	2.43	1.75	1.10
10	5	4	4	4	3	3	3	2	2
	1.28	.823	.823	1.37	.818	.818	1.10	.532	.532

*The sample size n is obtained by taking the largest integer less than or equal to the tabled value divided by p_0, i.e., $n=[D/p_0]$.
Producer's risk α is the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.
Consumer's risk β is the probability of accepting lots with unacceptable proportion of lot failing before specified time, p_1.

SECTION 2D

SEQUENTIAL LIFE TEST SAMPLING PLANS

2D1 SEQUENTIAL LIFE TEST SAMPLING PLANS

This part of the handbook describes the procedures for use in determining lot acceptability with sequential life tests. Two procedures are given: (1) a procedure when testing without replacement, and (2) another procedure when testing with replacement.

2D1.1 Use of Sequential Life Test Plans. To determine whether the lot meets the acceptability criterion with respect to average length of life, the applicable sampling plan shall be used in accordance with the provisions of section 2A, General Description of Life Test Plans, and those in this part of the handbook.

2D1.2 Drawing of Samples. All samples shall be drawn in accordance with paragraph 1A6.

2D1.3 Sample Plan Code Designation. The sample plan code designation shall be selected from table 2A-1.

2D2 SELECTING SEQUENTIAL LIFE TEST PLANS FOR DETERMINING LOT ACCEPTABILITY

2D2.1 Master Sampling Table. The master sampling table for sequential life test plans for determining acceptability of a lot is table 2D-1.

2D2.2 Obtaining the Sampling Plan. The sequential life test sampling plan consists of a sample size n, the acceptance line intercept h_0, the rejection line intercept h_1, and the common slope s of the two lines. The sampling plan is obtained from the master table.

2D2.2.1 Sample Size. The minimum number of units of product that shall be placed on test, r_0, when testing without replacement, is shown in the master table. When testing with replacement, the sample may be of any size. Increasing the sample size, in either sampling with or without replacement, will decrease the time required to reach a decision as to lot acceptability.

2D2.2.2 Acceptance Line Intercept. The acceptance line intercept h_0 is obtained from the master table by multiplying the entry corresponding to the sample plan code designation by the acceptable mean life θ_0.

2D2.2.3 Rejection Line Intercept. The rejection line intercept h_1 is obtained from the master table by multiplying the entry corresponding to the sample plan code designation by θ_0.

2D2.2.4 Slope of Decision Lines. The common slope s of the acceptance and rejection lines is obtained from the master table by multiplying the entry corresponding to the sample plan code designation by θ_0.

2D2.3 Acceptance Time. The acceptance time

$$h_0 + ks$$

where h_0 is the acceptance line intercept obtained in paragraph 2D2.2.2
- s is the slope of the decision lines as obtained in paragraph 2D2.2.4
- k is the number of failed units of product observed in the length of time that the life test has been in progress

shall be computed for $k = 0, 1, 2, \ldots$.

2D2.4 Rejection Time. The rejection time

$$h_1+ks$$

where h_1 is the rejection line intercept obtained in paragraph 2D2.2.3
s is the slope of the decision lines as obtained in paragraph 2D2.2.4
k is the number of failed units of product observed in the length of time that the life test has been in progress

shall be computed for $k=0, 1, 2, \ldots$. Negative values of the rejection time mean that more failures must occur before the rejection of the lot is allowed and hence may be disregarded.

2D3 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITHOUT REPLACEMENT

2D3.1 Total Life. The acceptability of a lot with respect to a sequential life test shall be judged by the quantity $v(t)$.

2D3.2 Computation. The following quantity shall be computed from the test results:

$$V(t)=\sum_{i=1}^{k} x_{i,n}+(n-k)t$$

where $x_{i,n}$ denotes the time of the i'th failure in a sample of size n
t denotes the length of time that the life test has been in progress
k denotes the number of failed units of product observed in time t
$V(t)$ denotes the total length of time survived by all units of product on test, failed and unfailed, up to time t.

2D3.3 Acceptability Criterion. Compare the quantity $V(t)$ with the acceptance time, mentioned in paragraph 2D2.3, and with the rejection time, mentioned in paragraph 2D2.4. If $V(t)$ is equal to or greater than h_0+ks, the lot meets the acceptability criterion; if $V(t)$ is less than or equal to h_1+ks, the lot does not meet the acceptability criterion; and if $h_1+ks<V(t)<h_0+ks$, the evidence is insufficient to reach a decision as to acceptability so the life test must continue and the above procedures repeated at a later time t.

2D4 LOT ACCEPTABILITY PROCEDURES WHEN TESTING WITH REPLACEMENT

2D4.1 Total Life. The acceptability of a lot with respect to a sequential life test shall be judged by the quantity $V(t)$.

2D4.2 Computation. The following quantity shall be computed from the test results:

$$V(t)=nt$$

where n denotes the number of units of product placed on test originally
t denotes the length of time that the life test has been in progress
$V(t)$ denotes the total length of time survived by all units of product, failed and unfailed, original units and replacement units, up to time t.

2D4.3 Acceptability Criterion. Compare the quantity $V(t)$ with the acceptance time, mentioned in paragraph 2D2.3, and with the rejection time, mentioned in paragraph 2D2.4. If $V(t)$ is equal to or greater than h_0+ks, the lot meets the acceptability criterion; if $V(t)$ is less than or equal to h_1+ks, the lot does not meet the acceptability criterion; and if $h_1+ks<h_0+ks$, the evidence is insufficient to reach a decision as to acceptability so the life test must continue and the above procedures repeated at a later time t.

2D5 CHOICE OF TIMES FOR DETERMINING ACCEPTABILITY

The procedures of sections 2D3 and 2D4 allow the acceptance of the lot if no failures are observed before time h_0/n has elapsed; rejection of the lot is allowed at any time $t>0$ but the number of failures must exceed $-h_1/s$. If decisions as to lot acceptability can be made continuously in time, the greatest savings in expected waiting time and the number of failures required for decision are realized over the procedures of Sections 2B and 2C. However, if acceptance of the lot has not been allowed at time h_0/n, computation of $V(t)$, mentioned in paragraphs 2D3.2 or 2D4.2, may be made periodically. The computation of $V(t)$ must follow each failure but may be made oftener in order to reduce the waiting time required for decision.

2D6 TRUNCATION OF SEQUENTIAL LIFE TESTS

The sequential life test, when testing without replacement, will terminate, at the latest, when all units of product placed on test have failed; the sequential life test, when testing with replacement, should not be allowed to run indefinitely but may be terminated by the procedures of this paragraph. In either case, the sequential life test may be terminated and the lot considered to have met the acceptability criterion if the number of failures is less than r_0, where the value of r_0 is obtained from table 2D–1, and if—

$$V(t) \geqq min(h_0+ks,\ sr_0)$$

where $V(t)$ and s are explained in paragraphs 2D2.2.4 and 2D3.2 or 2D4.2 and min $(h_0+ks,\ sr_0)$ means the lesser of either h_0+ks or sr_0. The sequential life test shall be terminated and the lot considered to have failed to meet the acceptability criterion if $V(t) \leqq h_1+ks$ or if the number of failures equals r_0 and $V(t) < sr_0$.

2D7 GRAPHICAL ACCEPTANCE PROCEDURES

2D7.1 Charts Made Before Start of Life Test. The acceptability procedures of Sections 2D2 to 2D6 may be drawn on a chart with the vertical axis representing $V(t)$ and the horizontal axis representing k (see example 2D–3 and fig. 2D–1). The acceptance line shall be drawn with vertical axis intercept equal to h_0 and with slope s. The rejection line shall be drawn with vertical axis intercept h_1 and with slope s. If the sequential test is to be truncated, a horizontal line shall be drawn at $V(t) = sr_0$ from the acceptance line to the point $(r_0,\ sr_0)$. From this point, a vertical line is drawn to the rejection line (see example 2D–4 and fig. 2D–2).

2D7.2 Plotting of Data. The life test data $V(t)$, as obtained by the procedures of paragraph 2D3.2 or 2D4.2, shall be plotted on the chart prepared in accordance with paragraph 2D7.1, by moving vertically as long as the next failure is being awaited, and moving horizontally by one unit (in k) at the occurrence of each failure. The life test continues until the plotted data touches one of the lines already drawn on the chart. If the plotted data touches the acceptance line or the horizontal line $V(t) = sr_0$ at a point to the left of $(r_0,\ sr_0)$, the lot meets the acceptability criterion. If the plotted data touches the rejection line or the vertical line below the point $(r_0,\ sr_0)$, the lot does not meet the acceptability criterion.

2D8 EXPECTED NUMBER OF FAILURES REQUIRED FOR DECISION

The expected number of failures required for a decision as to lot acceptability is dependent on the mean life of the lot. The master table gives the expected numbers of failures required when the mean life is equal to 0, θ_1, s (the mean life whose numerical value is equal to the slope of par. 2D2.2.4), and θ_0; and are denoted by $E_0(r)$, $E_{\theta 1}(r)$, $E_s(r)$, and $E_{\theta 0}(r)$, respectively. These values apply whether the testing is with or without replacement and assume that the decisions are made continuously in time. If $V(t)$ is computed periodically, the expected values may be exceeded.

2D9 EXPECTED WAITING TIME REQUIRED FOR DECISION

The expected waiting time required for a decision as to lot acceptability is dependent on the mean life of the lot and the number of units of product placed on test. When testing without replacement, the expected waiting times, when $\theta = 0$, θ_1, s, and θ_0, are given approximately by—

$$E_0(t) = 0$$
$$E_{\theta 1}(t) \cong \theta_1\ log_e\{n/[n - E_{\theta 1}(r)]\}$$
$$E_s(t) \cong s\ log_e\{n/[n - E_1(r)]\{$$
$$E_{\theta 0}(t) \cong \theta_0\ log_e\{n/[n + E_{\theta 0}(r)]\}$$

When testing with replacement, the expected waiting times are given by—

$$E_0(t)=0$$
$$E_{\theta_1}(t)=\theta_1 E_{\theta_1}(r)/n$$
$$E_s(t)=s\ E_s(r)/n$$
$$E_{\theta_0}(t)=\theta_0 E_{\theta_0}(r)/n$$

These values are based on the assumption that the decisions are made continuously in time so that if $V(t)$ is computed only periodically, the expected values may be exceeded.

Example 2D-1

Selection of Sequential Life Test Plan

Example: Find a sequential replacement procedure which will accept a lot with acceptable mean life $\theta_0=1{,}500$ hours, 95 percent of the time and will reject a lot with unacceptable mean life $\theta_1=300$ hours, 90 percent of the time. In this case $\theta_0=1{,}500$, $\theta_1=300$, $\alpha=0.05$, and $\beta=0.10$.

Solution: Since $\theta_1/\theta_0=300/1{,}500=0.200$, table 2A-1 gives sequential life test plan B-4 as the proper plan to be used. From table 2D-1, the following quantities are obtained: $h_0=\theta_0(h_0/\theta_0)=1{,}500(.5805)=870.75$ hours, $h_1=\theta_0(h_1/\theta_0)=1{,}500(.7453)=1{,}117.95$ hours, $s=\theta_0(s/\theta_0)=1{,}500(.4086)=612.9$ hours/failure, and minimum $n=r_0=12$. Substituting in the formula of paragraphs 2D4.2 and 2D4.3, the life test is continued as long as the inequality—

$$-1{,}117.95+612.9\ k>nt>870.75+612.9\ k$$

holds and is stopped as soon as the inequality does not hold. If 20 units of product are placed on test, the inequality may be written—

$$-55.90+30.64\ k>t>43.54+30.64\ k$$

If, at the time of stopping, t is less than the left-hand member of the inequality, the lot is considered to have failed to meet the acceptability criterion; if, at the time of stopping, t is greater than the right-hand member of the inequality, the lot is considered to have met the acceptability criterion.

Example 2D-2

Expected Number of Failures and Waiting Time

Example: Determine the expected number of failures and the expected waiting time required for a decision in the sequential life test plan of example 2D-1 if $n=20$ and the mean life of the lot is 0, θ_1, s, and θ_0 hours.

Solution: From table 2D-1, for sequential plan B-4,

$$E_0(r)=1.8 \text{ units of product}$$
$$E_{\theta_1}(r)=3.0 \text{ units of product}$$
$$E_s(r)=2.6 \text{ units of product}$$
$$E_{\theta_0}(r)=0.9 \text{ units of product}$$

and from the formulas of paragraph 2D9,

$$E_0(t)=0 \text{ hours}$$
$$E_{\theta_1}(t)=310(3.0)/20=46.5 \text{ hours}$$
$$E_s(t)=(612.9)(2.6)/20=79.7 \text{ hours}$$
$$E_{\theta_0}(t)=1{,}500(0.9)/20=67.5 \text{ hours}$$

Example 2D–3

Sequential Life Test Plan

Example: In the problem of example 2D–1, suppose that a sample of size 20 is placed on test. Units of product that fail are replaced immediately by new units of product drawn from the same lot. The life test is started at time $t=0$ and the first five failures occur at $x_{1,20}=25$ hours, $x_{2,20}=55$ hours, $x_{3,20}=70$ hours, $x_{4,20}=100$ hours, and $x_{5,20}=160$ hours, all times being measured from $t=0$.

(*a*) Verify that no decision has been reached by time $x_{5,20}$.

(*b*) Verify that if the sixth failure has not occurred at 196.74 hours, measured from $t=0$, the life test may be terminated at that time with acceptance of the lot.

Solution: The acceptance procedure, as described in section 2D7, is drawn in figure 2D–1. The plotted data is still within the two decision lines at time $x_{5,20}=160$ or $V(t)=20(160)=3{,}200$, but crosses the acceptance line when $k=5$ at time $t=43.54+30.64(5)=196.74$ since $t=V(t)/n$. Since the sixth failure has not yet occurred, the life test may be terminated at $t=196.74$ hours with the acceptance of the lot.

Remark: If the sixth failure had occurred at $t=225$ hours, the time saved by making decisions continuously in time is $225-196.74=28.26$ hours. Thus, if $V(t)$ is computed only after occurrence of a failure, the life test would have been prolonged 28.26 hours needlessly.

Example 2D–4

Truncated Sequential Life Test Plan

Example: In the problem of example 2D–1, suppose that the sequential life test plan is truncated and a sample of size 20 is placed on test at time $t=0$. The first 12 failures occur at: 25, 55, 70, 100, 160, 190, 200, 225, 235, 290, 320, and 335 hours. Verify that the lot does not meet the acceptability criterion.

Solution: The acceptance procedure, as described in section 2D7, is drawn in figure 2D–2. The plotted data crosses the vertical line below the point (r_0, sr_0) or (12, 7354.8) so the lot does not meet the acceptability criterion.

Remark: While the acceptance of the lot in example 2D–3 is made *between* failure times $x_{5,20}$ and $x_{6,20}$, rejection of the lot in example 2D–4 is made at the failure time $x_{12,20}$. This illustrates the point that acceptance of a lot can always be made between failure times, whereas rejection of a lot can only be made at a failure time.

Fig. 2D-1

EXAMPLE OF SEQUENTIAL LIFE TEST

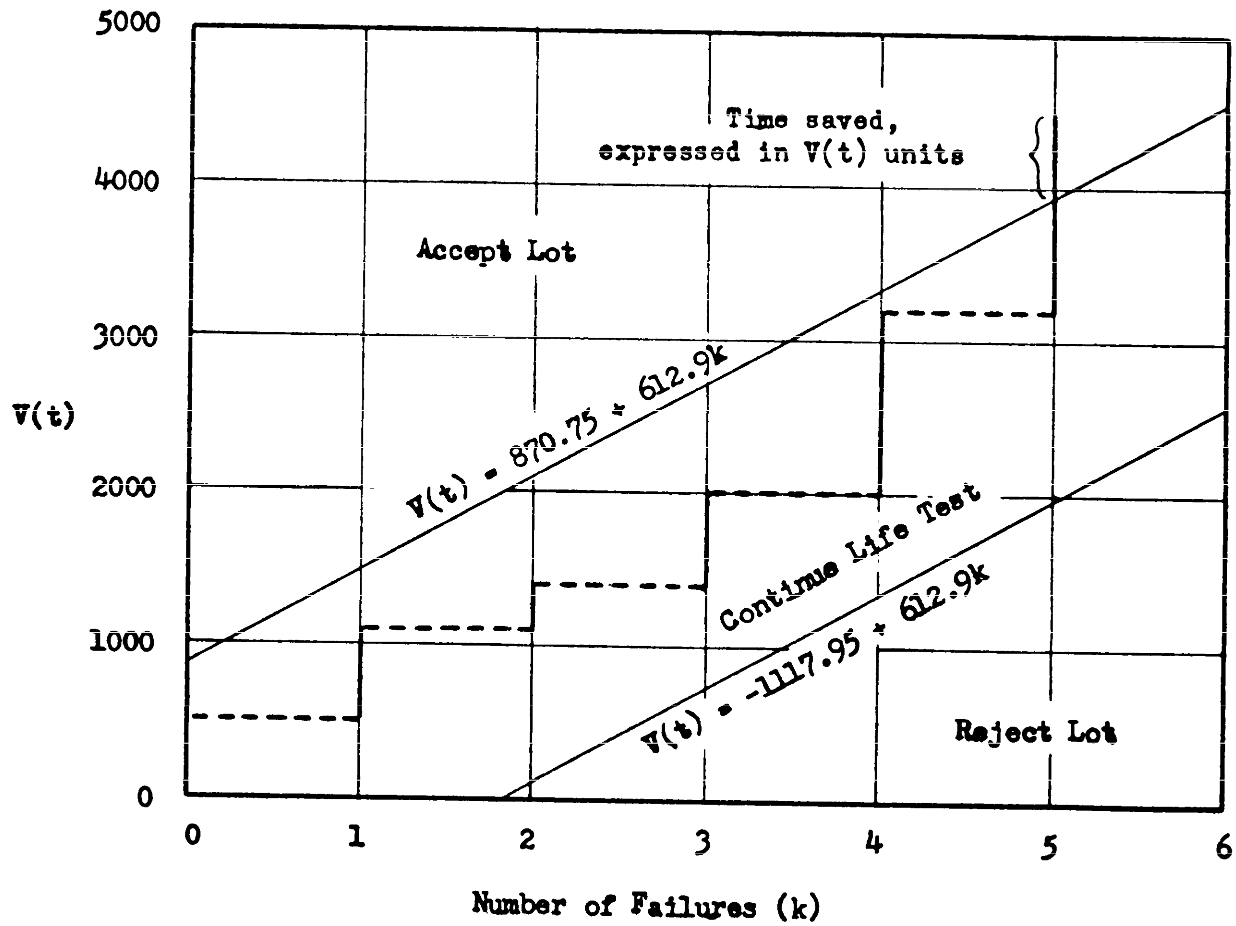

FIG. 2D-2

EXAMPLE OF TRUNCATED SEQUENTIAL LIFE TEST

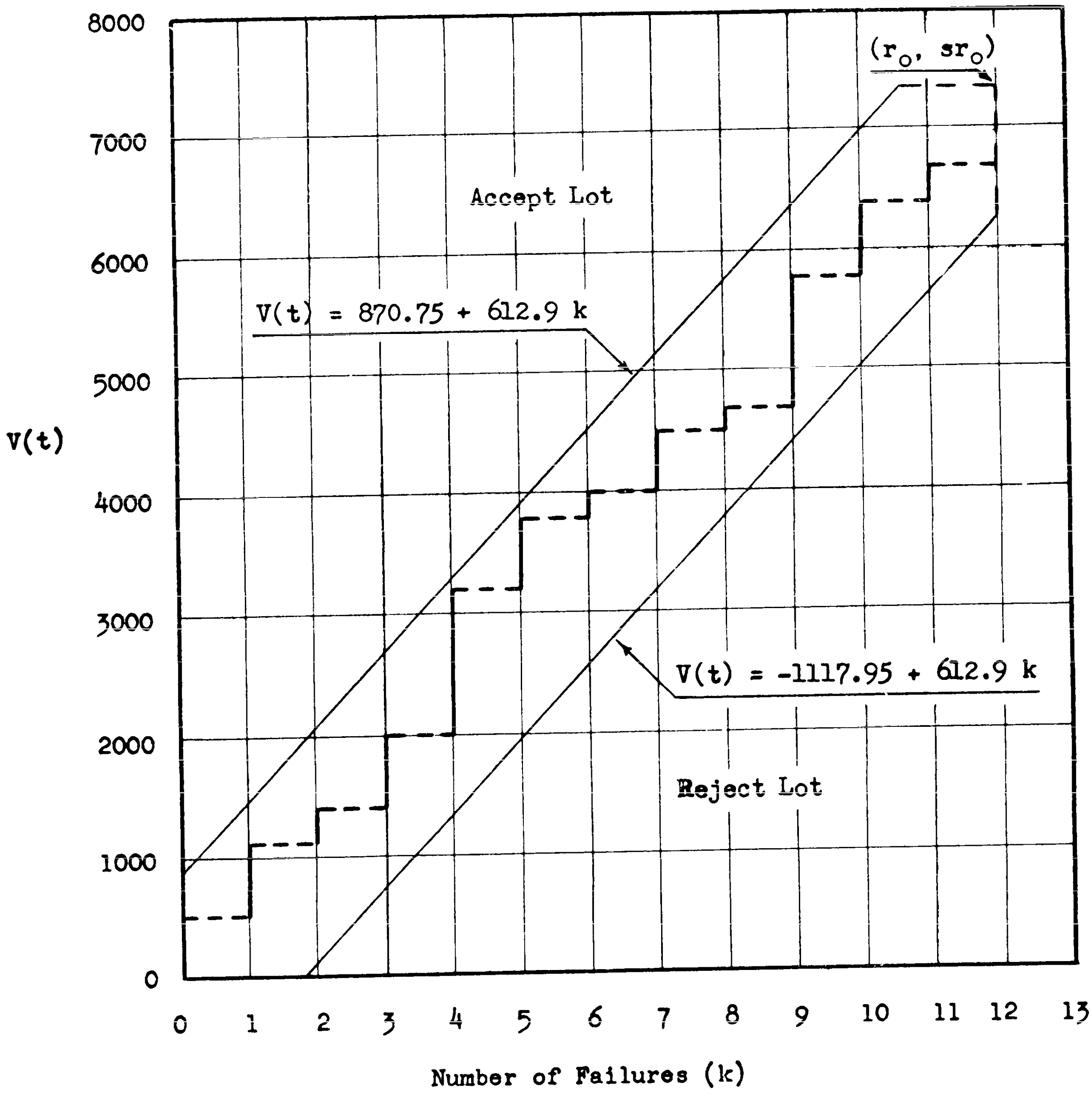

Table 2D-1(a)

Master Table for Sequential Life Tests

$\alpha = 0.01$

Code	r_0	h_0/θ_0	h_1/θ_0	s/θ_0	$E_0(r)$	$E_{\theta_1}(r)$	$E_s(r)$	$E_{\theta_0}(r)$
A-1	3	.0092	−0.0181	0.0222	0.8	0.8	0.3	0.0
A-2	6	.0906	−.1777	.1292	1.4	1.7	1.0	.1
A-3	9	.2048	−.4019	.2234	1.8	2.4	1.6	.3
A-4	12	.3215	−.6311	.2939	2.1	3.1	2.3	.4
A-5	15	.4367	−.8571	.3491	2.5	3.8	3.1	.7
A-6	18	.5483	−1.0762	.3934	2.7	4.6	3.8	.9
A-7	21	.6504	−1.2766	.4283	3.0	5.2	4.5	1.1
A-8	24	.7520	−1.4760	.4587	3.2	5.9	5.3	1.3
A-9	27	.8479	−1.6643	.4843	3.4	6.6	6.0	1.6
A-10	30	.9409	−1.8469	.5067	3.6	7.3	6.8	1.9
A-11	45	1.3522	−2.6541	.5848	4.5	10.5	10.5	3.2
A-12	60	1.7154	−3.3670	.6350	5.3	13.8	14.3	4.6
A-13	75	2.0330	−3.9904	.6696	6.0	17.0	18.1	6.0
A-14	90	2.3295	−4.5724	.6962	6.6	20.2	22.0	7.4
A-15	120	2.8477	−5.5894	.7336	7.6	26.4	29.6	10.4
A-16	150	3.3127	−6.5022	.7600	8.6	32.7	37.3	13.4
A-17	225	4.3142	−8.4679	.8020	10.6	48.3	56.8	21.1
A-18	300	5.1508	−10.1100	.8272	12.2	63.5	76.1	28.9

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2D-1(b)

Master Table for Sequential Life Tests

$\alpha = 0.05$

Code	r_0	h_0/θ_0	h_1/θ_0	s/θ_0	$E_0(r)$	$E_{\theta_1}(r)$	$E_s(r)$	$E_{\theta_0}(r)$
B-1	3	0.0506	−0.0650	0.0859	0.8	0.8	0.4	0.0
B-2	6	.2254	−.2894	.2400	1.2	1.6	1.1	.3
B-3	9	.4098	−.5261	.3405	1.5	2.3	1.9	.6
B-4	12	.5805	−.7453	.4086	1.8	3.0	2.6	.9
B-5	15	.7345	−.9430	.4576	2.1	3.7	3.3	1.2
B-6	18	.8842	−1.1352	.4972	2.3	4.3	4.1	1.6
B-7	21	1.0209	−1.3107	.5282	2.5	5.0	4.8	1.9
B-8	24	1.1495	−1.4757	.5538	2.7	5.6	5.5	2.3
B-9	27	1.2719	−1.6329	.5756	2.8	6.3	6.3	2.7
B-10	30	1.3916	−1.7866	.5948	3.0	6.9	7.0	3.0
B-11	45	1.9101	−2.4523	.6607	3.7	10.0	10.7	5.0
B-12	60	2.3620	−3.0325	.7024	4.3	13.1	14.5	7.0
B-13	75	2.7516	−3.5327	.7307	4.8	16.1	18.2	9.1
B-14	90	3.1217	−4.0079	.7530	5.3	19.2	22.1	11.2
B-15	120	3.7522	−4.8173	.7833	6.2	25.0	29.5	15.3
B-16	150	4.3314	−5.5610	.8053	6.9	31.0	37.1	19.7
B-17	225	5.5386	−7.1109	.8391	8.5	45.6	55.9	30.5
B-18	300	6.5773	−8.4444	.8600	9.8	60.4	75.1	41.6

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2D-1(c)

Master Table for Sequential Life Tests

$\alpha = 0.10$

Code	r_0	h_0/θ_0	h_1/θ_0	s/θ_0	$E_0(r)$	$E_{\theta_1}(r)$	$E_s(r)$	$E_{\theta_0}(r)$
C-1	3	0. 1059	−0. 1059	0. 1485	0. 7	0. 8	0. 5	0. 0
C-2	6	. 3488	−. 3488	. 3156	1. 1	1. 6	1. 2	. 4
C-3	9	. 5736	−. 5736	. 4111	1. 4	2. 2	1. 9	. 8
C-4	12	. 7760	−. 7760	. 4744	1. 6	2. 9	2. 7	1. 2
C-5	15	. 9597	−. 9597	. 5201	1. 8	3. 6	3. 4	1. 6
C-6	18	1. 1319	−1. 1319	. 5558	2. 0	4. 2	4. 1	2. 0
C-7	21	1. 2904	−1. 2904	. 5839	2. 2	4. 8	4. 9	2. 5
C-8	24	1. 4406	−1. 4406	. 6073	2. 4	5. 5	5. 6	2. 9
C-9	27	1. 5781	−1. 5781	. 6265	2. 5	6. 1	6. 3	3. 4
C-10	30	1. 7124	−1. 7124	. 6434	2. 7	6. 7	7. 1	3. 8
C-11	45	2. 3053	−2. 3053	. 7024	3. 3	9. 7	10. 8	6. 2
C-12	60	2. 8078	−2. 8078	. 7387	3. 8	12. 6	14. 4	8. 6
C-13	75	3. 2549	−3. 2549	. 7642	4. 3	15. 6	18. 1	11. 0
C-14	90	3. 6465	−3. 6465	. 7827	4. 7	18. 4	21. 7	13. 4
C-15	120	4. 3813	−4. 3813	. 8105	5. 4	24. 3	29. 2	18. 5
C-16	150	5. 0068	−5. 0068	. 8291	6. 0	29. 9	36. 5	23. 4
C-17	225	6. 3523	−6. 3523	. 8588	7. 4	43. 9	54. 7	36. 0
C-18	300	7. 5250	−7. 5250	. 8774	8. 6	48. 2	73. 6	49. 1

For explanation of the code, see par. 2A3.2 and table 2A-1.

Table 2D-1(d)

Master Table for Sequential Life Tests

$\alpha = 0.25$

Code	r_0	h_0/θ_0	h_1/θ_0	s/θ_0	$E_0(r)$	$E_{\theta_1}(r)$	$E_s(r)$	$E_{\theta_0}(r)$
D-1	3	0. 2878	−0. 1830	0. 2971	0. 6	0. 8	0. 6	0. 2
D-2	6	. 6609	−. 4202	. 4587	. 9	1. 5	1. 3	. 7
D-3	9	. 9701	−. 6167	. 5412	1. 1	2. 1	2. 0	1. 2
D-4	12	1. 2349	−. 7851	. 5930	1. 3	2. 7	2. 8	1. 8
D-5	15	1. 4651	−. 9314	. 6290	1. 5	3. 3	3. 4	2. 3
D-6	18	1. 6822	−1. 0694	. 6574	1. 6	3. 9	4. 2	2. 9
D-7	21	1. 8824	−1. 1967	. 6799	1. 8	4. 5	4. 9	3. 5
D-8	24	2. 0638	−1. 3120	. 6978	1. 9	5. 1	5. 6	4. 0
D-9	27	2. 2359	−1. 4215	. 7129	2. 0	5. 6	6. 3	4. 6
D-10	30	2. 4037	−1. 5281	. 7263	2. 1	6. 2	7. 0	5. 2
D-11	45	3. 1252	−1. 9868	. 7718	2. 6	9. 0	10. 4	8. 1
D-12	60	3. 7420	−2. 3789	. 8000	3. 0	11. 8	13. 9	11. 1
D-13	75	4. 2817	−2. 7220	. 8195	3. 3	14. 5	17. 4	14. 0
D-14	90	4. 7693	−3. 0320	. 8341	3. 6	17. 2	20. 8	17. 0
D-15	120	5. 6463	−3. 5895	. 8552	4. 2	22. 6	27. 7	23. 0
D-16	150	6. 4157	−4. 0786	. 8696	4. 7	27. 9	34. 6	29. 1
D-17	225	8. 0596	−5. 1237	. 8926	5. 7	41. 1	51. 8	44. 3
D-18	300	9. 4334	−5. 9971	. 9063	6. 6	54. 1	68. 9	59. 5

For explanation of the code, see par. 2A3.2 and table 2A-1.

APPENDIX 2D–1

In figure 2D–3, a comparison is shown in the expected number of failures required for a decision between a life test terminated at preassigned time when sampling with replacement plan of section 2C and that for a sequential plan of section 2D. Both plans have OC curves passing through the points ($\theta_0=1500$, $1-\alpha=0.95$) and ($\theta_1=300$, $\beta=0.10$).

In figure 2D–4, a comparison is shown in the expected waiting times required for a decision between a life test terminated at preassigned time when sampling with replacement plan and a sequential replacement plan. Both plans have sample size $n=20$ and have OC curves passing through the points ($\theta_0=1500$, $1-\alpha=0.95$) and ($\theta_1=300$, $\beta=0.10$).

The above comparisons are typical of the savings in time and number of failures required for a decision.

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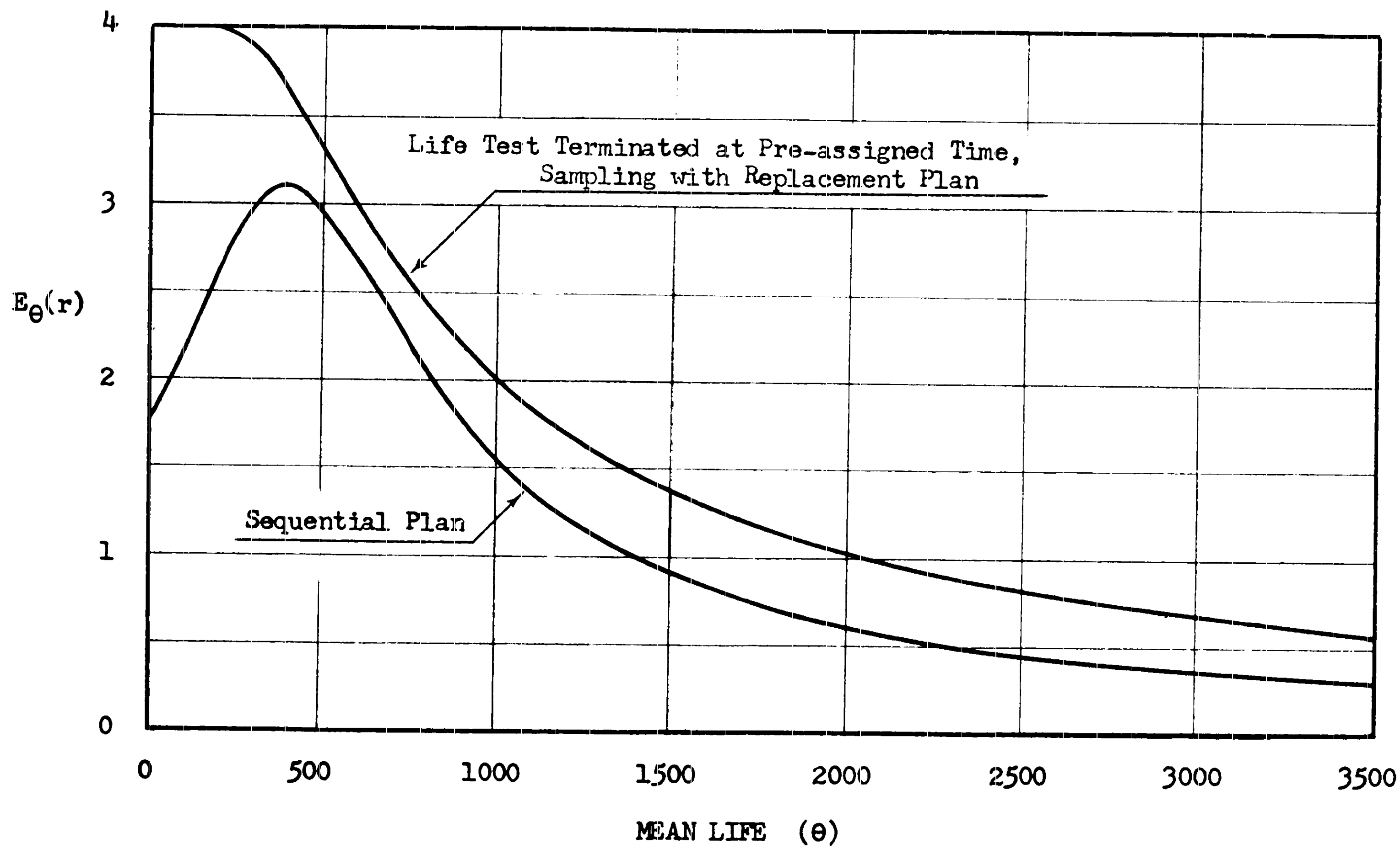
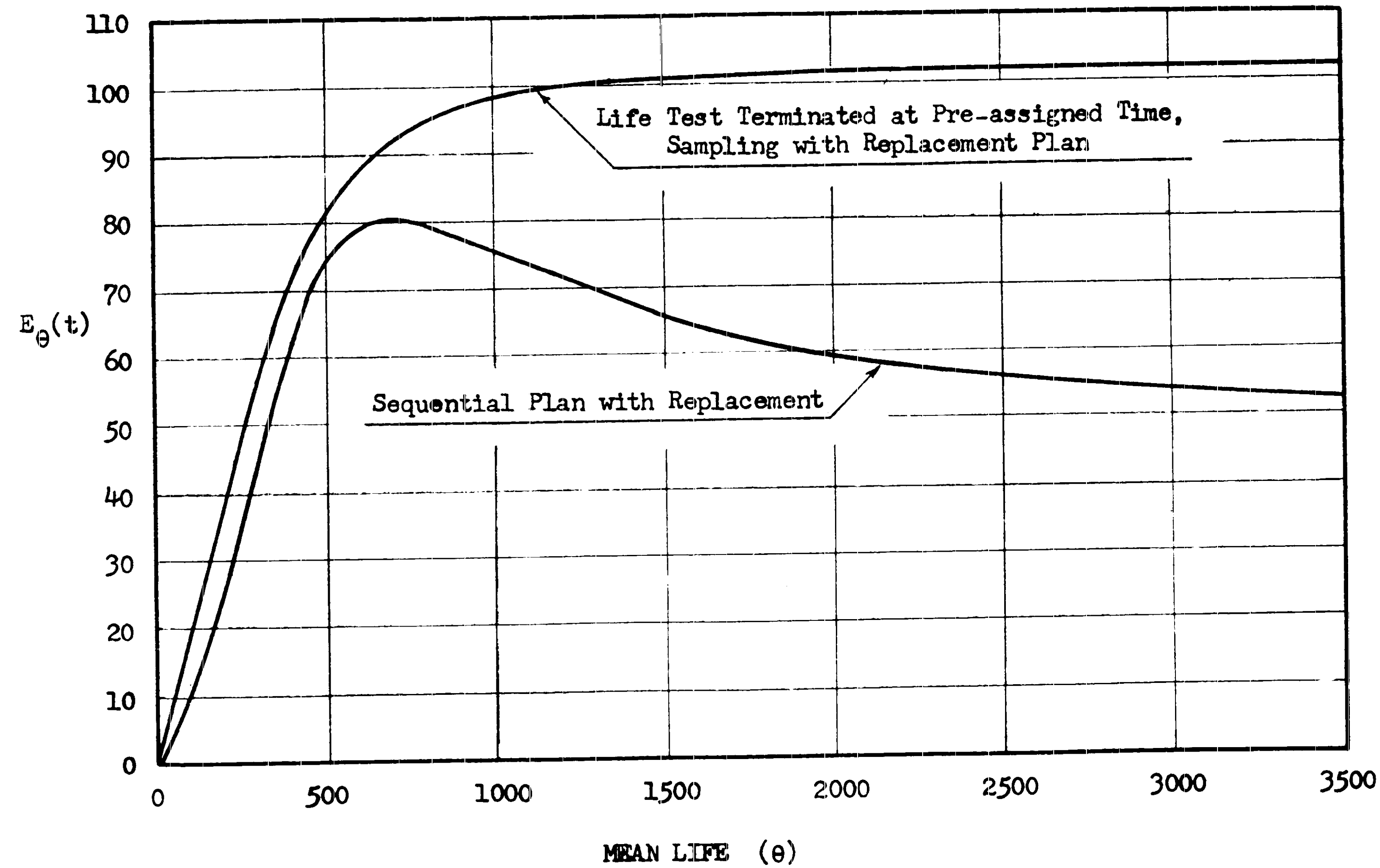
FIG. 2D-3

COMPARISON OF EXPECTED NUMBER OF FAILURES REQUIRED FOR DECISION FOR SEQUENTIAL AND LIFE TEST TERMINATED AT PRE-ASSIGNED TIME, SAMPLING WITH REPLACEMENT PLANS



The OC curve for each plan is such that $\theta_0 = 1500$, $\theta_1 = 300$, $\alpha = 0.05$, and $\beta = 0.10$.

COMPARISON OF EXPECTED WAITING TIME REQUIRED FOR DECISION FOR SEQUENTIAL AND LIFE TEST TERMINATED AT PRE-ASSIGNED TIME, SAMPLING WITH REPLACEMENT PLANS



The OC curve for each plan is such that $\theta_0 = 1500$, $\theta_1 = 300$, $\alpha = 0.05$, and $\beta = 0.10$.

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